A Cooperative Game Theory Approach for Cost Allocation in Complex Water Resource Systems

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SUMMARY

In order to obtain an optimal water resource planning and management, a fair and efficient rule of cost allocation among the different water users has a very important function. Starting from a correct assignment of costs it is possible to establish the basis to define the pricing policy that considers the principles of cost recovery and the adequate contribution for the different water uses as required by the Directive 2000/60/EC (EU, 2000).

The majority of cost allocation methods currently used in water resources system do not highlight the motivation of adopted criteria; so, we wonder why the users should accept an assignment which exceeds their opportunity cost or their willingness to pay (Young, 1985).

The main problem searching for a commonly accepted division of costs is how to share the costs in a fair and just way providing an adequate justification of criteria of the adopted methods.

Therefore, the aim of the research is to individuate an impartial and fair method for cost sharing for all the users who respect the principles of individual acceptability and general agreement and to argue for voluntary cooperation among the interested agents in order to maximize the efficiency of water resource management.

In this thesis we present a methodology of water services cost allocation based on Cooperative Game Theory, which can be a very usable instrument for the decision makers in order to elaborate pricing policies for water resources systems, according to the principles of the Directive 2000/60/EC.

The validation of the methodology has been realized for the water system Flumendosa-Capidano in Sardinia.
RIASSUNTO

Nella pianificazione e gestione ottimale dei sistemi di risorse idriche assume notevole importanza l’esigenza di ripartire i costi sostenuti tra i differenti utenti in maniera equa ed efficiente. A partire da una corretta assegnazione dei costi è possibile porre le basi per la definizione di una politica dei prezzi che tenga conto dei principi di recupero dei costi e di adeguato contributo per i differenti usi dell’acqua, come richiesto dalla Direttiva Quadro 2000/60/CE (EU, 2000).

La maggior parte dei metodi di ripartizione dei costi, attualmente utilizzati nell’ambito della gestione delle risorse idriche, ha il difetto di non porre in giusto rilievo la motivazione dei criteri adottati: ci si chiede, infatti, il perché gli utenti debbano accettare un’assegnazione che ecceda i loro costi opportunità o la loro disponibilità a pagare (Young, 1985).

Il problema principale, nella ricerca di una ripartizione condivisa dei costi, è quello di trovare una ripartizione che sia ritenuta equa e giusta fornendo un’adeguata giustificazione dei criteri posti alla base della legge di attribuzione.

Pertanto, l’obiettivo della ricerca è quello di individuare una ripartizione imparziale ed equa da parte di tutti gli utenti, che rispetti i principi di accettabilità individuale e di consenso generale e che favorisca, laddove risulti vantaggioso, la cooperazione volontaria tra i soggetti interessati al fine di massimizzare l’efficienza della gestione della risorsa idrica.

Nel presente lavoro di tesi si presenta una metodologia di ripartizione dei costi dei servizi idrici tra i vari utenti basata sulla Teoria dei Giochi Cooperativi. La ricerca si propone di definire uno strumento realmente fruibile dai decision maker per l’elaborazione delle politiche di prezzi nel settore delle risorse idriche, in accordo con i principi della Direttiva Quadro 2000/60/CE.

La validazione della metodologia è stata realizzata per lo schema idrico Flumendosa – Campidano in Sardegna.
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1. Introduction

1.1. Motivation of the research

1.1.1. The water services management

Water, the essential element for life and an indispensable factor for human development, apart from being considered all along as a social good, has assumed the characteristic of an economic good over the last few years. Consequently it is important to assign to it the right value considering its utilization, consumption, deterioration and possible pollution in order to encourage the user to use it sustainably.

In Italy and in the majority of European countries the assignation of grants, subsidies, refunds and contributions from the Government to the different enterprises, syndicates and operators of the water service has been a very common practice for years and almost taken for granted, which has quite often led to a depreciation of the resource.

However, new stricter European economic politics are in progress. In particular the recent European Directive 2000/60/EC \((EU, 2000)\) promoted the necessity of an economic analysis of water use, introducing the principle of recovery of costs in order to sustain the safeguard and the qualitative and quantitative improvement of bodies of water.

In this context, it is fundamental to fulfil a pricing policy strongly based on an accurate analysis of cost allocation among the users of a water resources system.
1.1.2. The European Directive 2000/60/EC

As we said before, over the last few years the European Community has paid particular attention to the problems of water resources management; in this context the directive 2000/60/EC aims to obtain the improvement of environmental conditions and in particular of the water bodies inside the Community territory.

The Directive established a reference in water politics and its major purpose is to improve the quality of the water resource, protecting and avoiding its deterioration for the future. To achieve this objective great importance has been attributed to water analysis, which in turn aims at: the quantification of the water resource, resources management, participation, information and public information and finally an economic analysis to guarantee the efficiency of the systems.

The problems linked to the definition of the criteria for the economically efficient management of the water systems represent one of the most important aspects of the European Directive. This matter is treated in particular in articles 5 and 9. The first one states that Member States shall ensure that for each river basin district, an economic analysis of water use is [...] undertaken and completed at the latest four years after the date of entry into force of this Directive [...]. In article 9 it is reported that Member States shall take account of the principle of recovery of the costs of water services, including environmental and resource costs, having regard to the economic analysis conducted [...] in accordance in particular with the polluter pays principle. Moreover, Member States shall ensure by 2010 that water-pricing policies provide adequate incentives for users to use water resources efficiently, [...] and there shall be a recovery of the costs of water services, for every different water uses, disaggregated into at least industry, households and agriculture, to the recovery of the costs of water services, based on the economic analysis conducted according to Annex III and taking account of the “polluter pays principle” (EU, 2000).

The introduction of the aforementioned economic indicators necessary to achieve the environmental objectives implies a recognition that water resources have an economic value, as ended resources. Therefore, economic analysis plays a decisive role in water resources management and in the design of new water price policies.

1.1.3. The cost allocation problem and the Cooperative Game Theory

The pricing methods currently used, which will be analyzed in details in Chapter 6, are found wanting, as they do not take into account the problem of our research question: we wonder why the final users have to accept an assignation which exceeds their opportunity cost or their willingness to pay (Young, 1985).

Therefore the main problem in defining a new rating politics is not the research of a general cost allocation rule, but rather how to share in an fair and just way the
sustained costs among the users. This means finding an impartial cost allocation for all the participants in a project, in order to promote and guarantee the collaboration among them and so the feasibility of a common project which enables the reduction of costs for all the beneficiaries.

As reported in Young’s article (1994) Cooperative Game Theory (hereafter CGT) provides the right instruments to analyze those situations in which it is important the research of a division mechanism that is efficient, fair and offers the appropriate incentives among the involved parts. Lemaire (1984) pointed out that the solution to a problem of cost allocation is equal to the determination of the value of a cooperative game. The cost division among different users can be seen, in fact, as a game in which it occurs to determine the right allocation among the different players. A cooperative game belongs to the mathematical science called Game Theory.

Game Theory, developed around the first half of the last century (Von Neumann & Morgenstern, 1944), studies the situations of conflicts in different fields and it researches competitive and/or cooperative solutions analyzing the individual decisions in situations where there are interactions among participants. In the scientific literature different cases of cost allocation that use CGT principles are present: the applicative fields are very different and also concern studies related to water resources (TVA, 1938; Young et al. 1982; Lippai & Heaney, 2000; Deidda et al., 2009).

Using CGT methods it is possible to “make explicit” the process of negotiation through mathematical formulas which implement properties that guarantee equity, fairness, justice and cooperation among players involved in a project, in order to aim a commonly acceptable solution.

Nevertheless, the application of CGT is limited essentially to economic and mathematical fields, avoiding the complexity and heterogeneity of engineering problems, i.e. water resources management. In fact, the calculation of a cooperative game requires an analysis of minimum cost of the system: this implies a process of optimization whose size increases exponentially in the function of the number of players. So the necessity to utilize adequate modelling instruments is the main obstacle to solve cost allocation problems in the case of complex systems (Deidda, 2009).

1.2. Objectives of the research

The main objective of the present study is the development of a methodology of cost allocation for water systems based on CGT that is able to contribute to the process of definition of water pricing according to the principles of the European Directive 2000/60/EC.

The methodology has a general aspect, suitable to the conditions of different river basins and compatible with the current instruments of water resources modelling.
The field of application can be extended to the solution of cost allocation problems both for local and regional systems.

Under a methodology based on the CGT it is possible to obtain a cost division among the users that can be shareable, can provide an adequate justification of adopted criteria and can favour cooperation among the interested subjects in order to maximize the efficiency of water resource management. This is a very important objective for Mediterranean water systems that are characterized by phenomena of water scarcity.

As a consequence, this thesis will attempt to make an original contribution to both the mathematical sciences and hydro-economic modelling.

Finally, this research belongs to the international project “Azioni Integrate Italia Spagna” (MIUR, 2007) between the department of “Ingegneria del Territorio” of the University of Cagliari and the department of “Ingeniería Hidráulica y Medio Ambiente” of the Polytechnic University of Valencia. The aim of the project is to develop a decisions supporting system to define measures to achieve the environmental and economic objectives required by the European Directive 2000/60.

### 1.3. Cost allocation methodology

The first phase of methodology is the identification of all the necessary aspects for the description and characterization of the water system to be studied; then the cooperative game defining the players and the typology of costs to be shared must be planned. Players can represent both the individual water user and groups of them, as in the case of users that belong to a single macro demand, for example irrigational and industrial syndicate, municipal centers, etc.

Then we can move on to the most important step, i.e. the definition of the characteristic function of the game, the basic element of the CGT. The function is formed by the set of minimum costs associated to all the possible coalitions of players, whose evaluation is carried out via the software WARGI (Sechi & Zuddas, 2000; Manca et al., 2004; Sechi & Sulis, 2009) that is based on a model of optimization specifically developed for water resources systems. The program enables the representation of a water system and it is also easy to input the required data (economic, hydrologic, hydraulic, infrastructural, etc.) for the functional definition of the system.

Once the characteristic function has been defined, it will be possible to solve the game applying the CGT methods.
1.4. Water system examined

The methodology has been verified in the application to the water scheme Flumendosa – Campidano situated in Sardinia.

The island is located in the centre of the western basin of the Mediterranean Sea and has a surface area of 24,000 km² with a population of 1,648,000 residents. The climate is prevalently Mediterranean, characterized by a long period of drought in summer and mild and rainy winters with isolated frosts.

After the application of the Regional Law n.19 of the 6.12.2006, in Sardinia the concept of “multi purpose water system” was introduced. This means the set of works for water supplying that, individually or as parts of a complex system, have the possibility to supply (directly or indirectly) more territorial areas or more different categories of users, contributing to an equalization of quantities and costs of supply (RAS, 2006b). The multi purpose system, whose management is entrusted to the Ente Acque della Sardegna – ENAS, supplies the wholesale water for the principal macro demands of the region: civil, irrigational and industrial.

The infrastructures which belong to the multi purpose regional system have been grouped into different “schemes” in relation to the use of the resource, allocating to the same scheme all the water works that, even if not directly interlinked, aim to satisfy the water needs of the same territory.

The Flumendosa-Campidano system consists of three multi purpose schemes and it supplies the water users of the central-southern zone of the island.

1.5. Structure of the document

The document is structured in nine chapters, including the present one with the introduction.

In Chapter 2 the regulations related to the water resources management in Europe, Italy and Sardinia is described, showing in details the reference rules adopted for the research.

In Chapter 3 the cost allocation problem is analyzed, examining principal allocation methods currently used, introducing the concept of “willingness to pay” and giving a possible solution to the introduced problem.

Chapter 4 is dedicated to the presentation of CGT with a brief initial description of the more general Game Theory; we expose in details the definitions, the principles and the solutions of CGT and, to conclude, we show some applications to complex systems.

The software WARGI is described in Chapter 5, where the different steps necessary to use it are analyzed; moreover the changes made to the program in order to better adapt it to the requirements of our research will be presented.
In Chapter 6 the Flumendosa-Campidano water system is described; furthermore we analyze the Sardinian water system and the different water sectors which compose it.

Chapter 7 is dedicated to the description of the cost allocation methodology and to its application to a simplified Flumendosa-Campidano system. Two water demand scenarios and two different approaches will be considered.

The application of the methodology to the complete Flumendosa-Campidano system is performed in Chapter 8, where the analysis of the results and the hypothesis of a new rating structure for the analyzed system are proposed.

The thesis ends with Chapter 9 in which some possible future developments for the research are presented and the conclusions and the original contributions provided by this work are summarized.

Moreover there are two final appendixes with the first and the last chapters in Italian.
2. Reference regulations

2.1. European regulations

2.1.1. Directive 2000/60/EC


The European Union (EU) has established a Community framework for water protection and management by issuing Directive 2000/60/EC (often mentioned as Water Framework Directive, hereafter WFD). Its main purpose is to maintain and, where necessary, to improve the quantitative and qualitative status of water bodies in the Community area. The WFD provides, among other things, for the identification and analysis of European waters on the basis of individual river basin districts, and the adoption of management plans and programmes of measures appropriate for each body of water.

This Directive provides that each Member State shall face waters' protection from the “river basin”; reference territorial unit to manage such basin is indicated in “river basin district”, an area of land and sea, made up of one or more neighbouring river basins together with their associated groundwaters and coastal waters. In every river basin district each Member State shall carry out:

- an analysis of district's characteristics;
- an exam of the impact of human activities on surface and groundwater;
- an economical analysis of water usage.
For every district there shall be a program of measures that has to control made
analysis and environmental objectives fixed from the Directive. These objectives
should be set to ensure a “good status” of all waters within 2015 (except in cases
clearly mentioned in this Directive). Programs of measures are indicated in
Management Plans that Member States have to establish for every river basin and
represent a programming instrument to reach objectives fixed from the Directive.

Article 5 of the Directive establishes that Member States shall carry out, for every
district, analyses of physical characteristics of districts, of human activities and the
economic analysis of water usage within four years from the implementation of
the Directive. Analyses and studies shall be reviewed after thirteen years and
subsequently after six years. Starting from 2010 Member States will need to
guarantee that water price policy stimulates consumers to use water resources in
an efficient way and guarantee for different sectors a cost recovery of water
service, including environmental and resource costs.

The introduction of an economic analysis of water resources can be considered the
most innovative aspect of this Directive. In practice, as stated before, article 5
states that Member States shall ensure that for each river basin district, an economic
analysis of water use is [...] undertaken and completed at the latest four years after the date
of entry into force of this Directive [...]. Furthermore, in article 9 it is reported that
Member States shall take account of the principle of recovery of the costs of water services,
including environmental and resource costs, having regard to the economic analysis
conducted [...] in accordance in particular with the polluter pays principle. Moreover,
Member States shall ensure by 2010 that water-pricing policies provide adequate
incentives for users to use water resources efficiently, [...] and there shall be a recovery of
the costs of water services, for every different water uses, disaggregated into at least
industry, households and agriculture, to the recovery of the costs of water services, based
on the economic analysis conducted according to Annex III and taking account of the
“polluter pays principle” (EU, 2000). In the following paragraphs economical
references present on Directive 2000/60/EC are reported.

2.1.1.1. Preliminary Considerations

Paragraph 1
Water is not a commercial product like any other but, rather, a heritage which must be
protected, defended and treated as such.

Paragraph 11
[...] the Community policy on the environment is to contribute to pursuit of the objectives
of preserving, protecting and improving the quality of the environment, in prudent and
rational utilisation of natural resources, and to be based on the precautionary principle and
on the principles that preventive action should be taken, environmental damage should, as
a priority, be rectified at source and that the polluter should pay.
Paragraph 12

[...] in preparing its policy on the environment, the Community is to take account of available scientific and technical data, environmental conditions in the various regions of the Community, and the economic and social development of the Community as a whole and the balanced development of its regions as well as the potential benefits and costs of action or lack of action.

Paragraph 36

It is necessary to undertake analyses of the characteristics of a river basin and the impacts of human activity as well as an economic analysis of water use. The development in water status should be monitored by Member States on a systematic and comparable basis throughout the Community. This information is necessary in order to provide a sound basis for Member States to develop programmes of measures aimed at achieving the objectives established under this Directive.

Paragraph 38

The use of economic instruments by Member States may be appropriate as part of a programme of measures. The principle of recovery of the costs of water services, including environmental and resource costs associated with damage or negative impact on the aquatic environment should be taken into account in accordance with, in particular, the polluter-pays principle. An economic analysis of water services based on long-term forecasts of supply and demand for water in the river basin district will be necessary for this purpose.

2.1.1.2. Article 5. Characteristics of the river basin district, review of the environmental impact of human activity and economic analysis of water use

1. Each Member State shall ensure that for each river basin district or for the portion of an international river basin district falling within its territory:

   - an analysis of its characteristics,
   - a review of the impact of human activity on the status of surface waters and on groundwater, and
   - an economic analysis of water use

is undertaken according to the technical specifications set out in Annexes II and III and that it is completed at the latest four years after the date of entry into force of this Directive.

2. The analyses and reviews mentioned under paragraph 1 shall be reviewed, and if necessary updated at the latest 13 years after the date of entry into force of this Directive and every six years thereafter.


2.1.1.3. Article 9. Recovery of costs for water services

1. Member States shall take account of the principle of recovery of the costs of water services, including environmental and resource costs, having regard to the economic analysis conducted according to Annex III, and in accordance in particular with the polluter pays principle.

Paragraph 38 of Article 2 defines «water services» as:

all services which provide, for households, public institutions or any economic activity:

(a) abstraction, impoundment, storage, treatment and distribution of surface water or groundwater;

(b) waste-water collection and treatment facilities which subsequently discharge into surface water.

Moreover Article 9 follows with these statements

Member States shall ensure by 2010:

- that water-pricing policies provide adequate incentives for users to use water resources efficiently, and thereby contribute to the environmental objectives of this Directive,

- an adequate contribution of the different water uses, disaggregated into at least industry, households and agriculture, to the recovery of the costs of water services, based on the economic analysis conducted according to Annex III and taking account of the polluter pays principle.

Member States may in so doing have regard to the social, environmental and economic effects of the recovery as well as the geographic and climatic conditions of the region or regions affected.

2. Member States shall report in the river basin management plans on the planned steps towards implementing paragraph 1 which will contribute to achieving the environmental objectives of this Directive and on the contribution made by the various water uses to the recovery of the costs of water services.

2.1.1.4. Article 11. Programme of measures

1. Each Member State shall ensure the establishment for each river basin district, or for the part of an international river basin district within its territory, of a programme of measures, taking account of the results of the analyses required under Article 5, in order to achieve the (environmental) objectives established under Article 4. Such programmes of measures may make reference to measures following from legislation adopted at national level and covering the whole of the territory of a Member State. Where appropriate, a Member State may adopt measures applicable to all river basin districts and/or the portions of international river basin districts falling within its territory.

2. Each programme of measures shall include the “basic measures” specified in paragraph 3 and, where necessary, “supplementary” measures.
3. “Basic measures” are the minimum requirements to be complied with and shall consist of:

[...]

(b) measures deemed appropriate for the purposes of Article 9;

[...].

2.1.1.5. **Annex III. Economic Analysis**

The economic analysis shall contain enough information in sufficient details (taking account of the costs associated with collection of the relevant data) in order to:

(a) make the relevant calculations necessary for taking into account under Article 9 the principle of recovery of the costs of water services, taking account of long term forecasts of supply and demand for water in the river basin district and, where necessary:

- estimates of the volume, prices and costs associated with water services,
- estimates of relevant investment including forecasts of such investments;

(b) make judgements about the most cost-effective combination of measures in respect of water uses to be included in the programme of measures under Article 11 based on estimates of the potential costs of such measures.

2.1.1.6. **Annex VI. Lists of measures to be included within the programmes of measures**

**PART B**

The following is a non-exclusive list of supplementary measures which Member States within each river basin district may choose to adopt as part of the programme of measures required under Article 11, paragraph 4:

(i) legislative instruments
(ii) administrative instruments
(iii) economic or fiscal instruments

[...]

2.1.1.7. **Annex VII. River Basin Management Plans**

A. River basin management plans shall cover the following elements:

1. a general description of the characteristics of the river basin district required under Article 5 and Annex II.

[...]

6. a summary of the economic analysis of water use as required by Article 5 and Annex III;
7. a summary of the programme or programmes of measures adopted under Article 11, including the ways in which the objectives established under Article 4 are thereby to be achieved; 

[...] 

7.2. a report on the practical steps and measures taken to apply the principle of recovery of the costs of water use in accordance with Article 9; 

[...] 

2.1.1.8. **Deadlines**

Table 1 reports the Directive deadlines that every Member State has to make reference.

<table>
<thead>
<tr>
<th>DEADLINES</th>
<th>FULFILMENTS PROVIDED BY DIRECTIVE 2000/60/EC</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 December 2000</td>
<td>Implementation of the Directive (Article 22)</td>
</tr>
</tbody>
</table>
| 22 December 2003| Laws, regulations and administrative provisions necessary to comply with the Directive come into effect (Article 24).  
Identification of the appropriate competent authority (Article 3). |
| 22 June 2004    | Member States shall provide the Commission with a list of their competent authorities (Article 3).            |
| 22 December 2004| For each river basin district a complete analysis of surface water and groundwater characteristics, a review of the impact of human activity, and an economic analysis of water use (Article 5).  
Establishment of a register or registers of all protected areas (Articles 6 and 7). |
| 22 December 2005| In the absence of criteria adopted under paragraph 2 at Community level, Member States shall establish appropriate criteria at the latest five years after the date of entry into force of the Directive (Article 17, comma 4).  
In the absence of criteria adopted under paragraph 4 at national level, trend reversal shall take as its starting point a maximum of 75% of the level of the quality standards set out in existing Community legislation applicable to groundwater. (Article 17, comma 5). |
<table>
<thead>
<tr>
<th>DEADLINES</th>
<th>FULFILMENTS PROVIDED BY DIRECTIVE 2000/60/EC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>22 December 2006</strong></td>
<td>Establishment of programmes for the monitoring of water status in order to establish a coherent and comprehensive overview of water status within each river basin district (Article 8). Publication of and consultation on a timetable and on a work programme for the production of the plan (Article 14). In the absence of agreement at Community level, for substances included in the first list of priority substances (Article 16), Member States shall establish environmental quality standards for these substances for all surface waters affected by discharges of those substances, and controls on the principal sources of such discharges (Article 16).</td>
</tr>
<tr>
<td><strong>22 December 2007</strong></td>
<td>Public information and consultation on an interim overview of the significant water management issues identified in the river basin. (Article 14).</td>
</tr>
<tr>
<td><strong>22 December 2008</strong></td>
<td>Member States shall allow at least six months to comment in writing on those documents in order to allow active involvement and consultation (Article 14).</td>
</tr>
<tr>
<td><strong>22 December 2009</strong></td>
<td>The drafting of a programme of measures for every basin district in order to achieve environmental objectives (Article 11) Predisposition and publication, for every district basin, of a management plan which includes the individuation of environmental objectives for every surface or subterranean water body and a summary of programmes of measures adopted to achieve such objectives (Article 13).</td>
</tr>
<tr>
<td><strong>2010</strong></td>
<td>Implementation of policies for a correct recovery of costs of water services (Article 9).</td>
</tr>
<tr>
<td><strong>22 December 2012</strong></td>
<td>Efficacy of programmes of measures in every river basin district to achieve environmental objectives (Article 11). Submission of an interim report describing progress in the implementation of the planned programme of measures (Article 15).</td>
</tr>
<tr>
<td><strong>22 December 2015</strong></td>
<td>Achievement of environmental objectives (Article 4).</td>
</tr>
<tr>
<td><strong>22 December 2015 and every 6 years for the following years</strong></td>
<td>Review and updating of plans (Articles 13, 14 and 15).</td>
</tr>
</tbody>
</table>

*Table 1. Deadline of Directive*
At the present moment, Italy is by far the country where the majority of delays, specially referred to economic analysis, are registered as reported in a recent workshop related to the accomplishment of WFD (Gruppo 183, 2009)

2.2. Italian Regulations

2.2.1. National legislation on water resources

Until the issuing of the legislative decree 152 of 2006 Italian legislation on water resources was based on five legislative orders:

- Royal Decree of 11 December 1933, n° 1775 (RD, 1933);
- Law of 10 May 1976, n° 319 (Merli Law) (Law, 1976);
- Law of 18 May 1989, n° 183 (Defence of Soil Law) (Law, 1989);
- Law of 5 January 1994, n° 36 (Galli Law) (Law, 1994);

The first Legislative Act on water resources was the Royal Decree 1775/33 where the principle of water as a public resource was declared for the first time and whose regulation was subordinated to the Public Administration. In this Act there was still an ancient view of water resources to be defended, but at the same time, it was necessary to exploit them through convenient infrastructures: so there was a gap in the concept of limited resource, to be conserved, to be protected and defended. The Decree established, moreover, suitable authorities, now recognizable as “Consorzi di Bonifica” (land-reclamation syndicates), entitled with a concession of water use for irrigational objectives.

In 1976 the first national regularization of discharges on water bodies “Norme per la tutela delle acque dall’inquinamento” (Rules to protect waters from pollution) was promulgated. It is known as Merli Law, after the name of its author, who defined it as a “police law”. Such a law was introduced in order to avoid an increase in pollution, establishing the basis for an effective defence of waters and soil, and limiting and controlling the principle sources of pollution and emission.

The first law in management and planning of water resources matters was the 183 in 1989: “Norme per il riassetto organizzativo e funzionale della difesa del suolo” (Rules for organizational and functional readjustment of soil defence). This law had the power to assure soil defence, waters recovery and the management of water for economic and social development and the concerned safeguard of environmental aspects. For this reason, a subdivision of the national territory in River Basins classified as National, Interregional and Regional relief was planned, in which a new body was created: the so-called Autorità di Bacino (Basin Authorities). These was given the task of submitting the Piano di Bacino (Basin Plan), defined as
“cognitive, normative and operative instrument through which actions and rules of use have been planned and programmed, in order to safeguard, defend and valorise the soil and the direct usage of waters, on the basis of physical and environmental characteristics of the relevant territory.

Subsequently in 1994 Law 36 was issued, known also as Galli Law, introduced to solve the massive fragmentation of the national water service for urban and civil use and to rationalize the legal situation. It proposed a unitary management of the cycle of depuration, distribution and drainage system. Besides clarifying the public nature of surface water and groundwater, whose usage for human consumption was declared a priority respect to others, the law introduced for the first time the concept of limited resource to use according to principles of solidarity. Another innovation was the identification of the so-called “Ambiti Territoriali Ottimali” (ATO) (Optimal territorial field), where an integrated management of drinking water service with a separation between property and management had to be guaranteed. To this end the law provided for an institution in every ATO of the Autorità d’Ambito (AATO), an institutional body charged with the task of controlling the managing subject of the “Servizio Idrico Integrato - SII” (integrated water service). Lastly there was an elaboration of a method to a reference rate in which it was to define new water rates for civil customers aimed to the principle of covering costs.

Another legislative act in water resources matters is Decree 152/99 “Disposizioni sulla tutela delle acque dall'inquinamento dall'inquinamento e recepimento della direttiva 91/271/EC e 91/676/CE” (Provisions about protection of waters from pollution and acknowledgement of the directive 91/271/EC, about urban wastewater treatment, and of the directive 91/676/EC, regarding the protection of waters against pollution caused by nitrates) which tried to rationalize and bring Italian regulations on the protection of water bodies up to date. The decree proposed to define the general subject to safeguard surface waters, marine and groundwater, in order to achieve objectives of prevention and reduction of pollution, improving the status of water bodies and sustainability for resources, keeping drinking resources a priority. The most relevant characteristics are the protection of quantitative and qualitative aspects in every river basin, individuation of environmental qualitative objectives which refer to definition of limits of discharge and the predisposition of measures for recovery and protection of water bodies. The administrative instrument used in the decree is the Piano di Tutela (Plan of Water Protection) in which environmental objectives, intervention and measures of pollution prevention are individuated.

Legislative provisions represented an important benchmark for defence, safeguard and planning of water resources in Italy. At the moment, except Royal Decree 1775/1933, such provisions are abrogated from Decree 152/2006 even though several concepts present in the Decree are in force.
2.2.2. Environmental Decree 152/2006

Legislative Decree 152/2006 “Norme in materia ambientale” (Rules of environmental subjects) (DL, 2006) represents the first national text about the environment. It is a normative corpus which consists of 318 articles which try to rationalize the environmental legislation analyzing five fundamental sectors:

- procedures to evaluate environmental strategy, to evaluate environmental impact and to integrated environmental authorization;
- defence of soil, fight against desertification, protection of waters against pollution and management of water resources;
- management of waste materials and drainage;
- defence of air and reduction of emissions on the atmosphere;
- environmental damage.

In water resources the Decree adopts the WFD and it specifically targets the following objectives:

- to amplify water protection, and surface water or groundwater;
- to reach the status of “good” for all the waters within the 31 December 2015;
- to manage water resources on the basis of territorial unity of reference as the drainage basin;
- to recognize all water services the right price which represent their real economic cost;
- to acquaint citizens with adopted choices.

Taking up what was provided by the Law 183/1989, national territory is divided into river basin districts (Figure 1), which in turn are divided into subunits as river basins. As regards Sardinia, the island represents a unique river basin formed by a unique basin which corresponds with regional territory.

According to the Decree, economic analysis and the consequent politic rates should be defined by applying, on one hand, principles of total recovery for costs of water service, considering financial, environmental and resource costs, and on the other, “the polluter principle”. It is important to follow the objective to make the customer afford his costs linked to the consumption of resources: for this reason in 2010 water service pricing policy will have to be established in order to guarantee a correct use of water and to contribute the achievement the WFD’s objectives. At any rate, by applying this principle it is possible to consider the social, environmental and economic repercussions of costs recovery, together with the geographic and climatic conditions of the individual regions. Integral covering of costs service represents a guide principle to be achieved where possible.
The new decree confirms what was established by the Galli Law, in terms of the drinking water cycle with the reaffirmation of ATO, SII and AATO, on which watches Comitato Nazionale per la Vigilanza sulle Risorse Idriche - CoNViRi (National Committee for the Water Resources Vigilance).

In pursuance of WFD, the law dissolves the Basin Authorities and institutes the District Basin Authority, giving it new responsibilities in order to achieve, according to deadlines, environmental objectives and parameters and an equilibrium of territory as established by Community Normative.

The District Basin Authority provides that there is a Piano di Bacino Distrettuale (District Basin Plan), a document filled by different parts like the Management Plan, which represents the institutional informative document as far as soil defence, safeguard of water and water resources management are concerned.

As reported in Annex 4 – Part A of Annex in the third part of the decree, the Management Plan includes the following elements:
- general description of characteristics of river basin district;
- a summary of significant pressures and impact of human activity on the status of water;
- cartographic specification and representation of protected areas;
- a list of environmental objectives for bodies of water;
- a summary of economic analysis on water use;
- a summary of programs of measures adopted, as:
  - synthesis of necessary measures to adopt community normative on water defence;
  - a report on the practical steps and measures taken to apply the principle of recovery of the costs of water;
  - a register of any more detailed programmes and management plans for the river basin district to be dealt with.

2.3. Sardinia regional regulations

In the present thesis the following Sardinia regional regulations on water resources management will also be considered.

2.3.1. Piano Stralcio di Bacino Regionale per l’Utilizzo delle Risorse Idriche - Regional River Basin Plan for Water Resources Use

The Regional Management River Basin Plan for Water Resources Use (PSURI) of Sardinia (RAS; 2006a), is a part of the bigger River Basin Plan required by the Law 183/1989. The objective of this document is to define structural and managing interventions aimed at obtaining a balance accounts between supply and demand on a regional level, according to economic and environmental limits of sustainability imposed by national and community rules.

PSURI acquire information about estimation of available resources, typology and quantification of needs of different water uses; it defines a map of infrastructures and hydraulic structures of regional water service and proposes infrastructural investments, estimating selection activities and technical and financial feasibility.

In the document there is an analysis about production cost of resource, id est, as shown in Chapter 1 of Study 5, “unitary cost of water production taken by surface resources for multiple uses as it can be determined on (theoretical) condition of entrepreneurial and unitary management of the system”. Such value “represents an element of reference to consider on following actions that Region (Sardinia) shall
undertake on achieving the right allocation of the production/use costs of resource, among user”.

To define production costs of resource it was necessary, in advance, “to estimate costs to support in order to organize in an efficient and efficacious way operative activities and works which constitute the management”. Therefore, they have analyzed different items of cost linked to regional water systems and, more specifically:

- employees costs;
- costs of energy;
- other costs (materials, informative system, parking etc.);
- costs of supplementary maintenance (SM);
- costs of routine maintenance (RM);
- costs of maintenance of capital.

For the present survey they have considered the analysis of Paragraph 1.3 of Study 5 and they refer to the calculation of maintenance costs of hydraulic service, here reported.

**Costs of supplementary maintenance**

Interventions of supplementary maintenance to which is referred are those programmed and constituted from all activities of revision, substitution of works, machines, electric and/or hydraulic systems, single mechanic components, operations, carpentry, aimed at contrasting or eliminating ageing or wear, and aimed at maintaining an efficiency status and full functions existing works during all the period of useful life and they are not available to extend such life.

This category of costs refers to the cost of investment of the infrastructure considering coefficients in relation with the different typology of work:

\[
C_{MS} = C_{I} \cdot c_{MS}
\]

where

- \(C_{MS}\): cost of yearly supplementary maintenance;
- \(C_{I}\): cost of investment (inclusive of general costs, technical expenses and VAT);
- \(c_{MS}\): coefficient of supplementary maintenance.

Coefficients of supplementary maintenance adopted are the result of data traced in books and of information available by Operators working on regional water systems (Table 2).
Table 2. Coefficients of supplementary maintenance

Costs of routine maintenance

The activity of routine maintenance is based on interventions aimed at maintaining a good status of preservation and protecting technique efficiency of works and facilities to guarantee normal efficacy. Therefore, the cost of ordinary and programmed activities from third parties to maintain building works through a contract and the cost of materials used from the domestic staff during maintenance activity have to be included in this item.

Maintenance costs, which, as shown above, do not include costs of domestic staff used employed for ordinary maintenance, are estimated for typology of works according to the following criteria:

- yearly unitary cost of maintenance for every work as regards reservoirs, diversion dams, tanks, divisors, pumping;
- yearly cost for kilometres concerning pipes, canals, tunnels.

Estimation of such parameters is obtained on the basis of technique characteristics of single works. The following Table 3 reports, for typology of work, utilized parameters.

<table>
<thead>
<tr>
<th>Kind of work</th>
<th>Unitary cost [€]</th>
<th>Cost per km [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dams</td>
<td>180.000</td>
<td></td>
</tr>
<tr>
<td>Diversion dams</td>
<td>10.000</td>
<td></td>
</tr>
<tr>
<td>Tanks and divisors</td>
<td>3.500</td>
<td></td>
</tr>
<tr>
<td>Pipes</td>
<td></td>
<td>4.500</td>
</tr>
<tr>
<td>Canals</td>
<td></td>
<td>8.000</td>
</tr>
</tbody>
</table>
Table 3. Adopted routine maintenance parameters

<table>
<thead>
<tr>
<th>Kind of work</th>
<th>Unitary cost [€]</th>
<th>Cost per km [€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnels</td>
<td></td>
<td>500</td>
</tr>
<tr>
<td>Pump stations &lt;=1.000kw</td>
<td>8.000</td>
<td></td>
</tr>
<tr>
<td>Pump stations &lt;=3.000 kw</td>
<td>12.000</td>
<td></td>
</tr>
<tr>
<td>Pump stations &gt;3.000 kw</td>
<td>15.000</td>
<td></td>
</tr>
</tbody>
</table>

**Investment cost of existing infrastructures**

In order to estimate the costs of supplementary maintenance, it is necessary to understand the opening value of water infrastructures for which cost functions have been adopted.

It is appropriate to show that realization costs of works reported here below include expenses related only to works and such costs do not include general costs, technical expanses and VAT.

**Dams**

In Figure 2 the function relative to realization cost of a concrete dam and in Figure 3 the function relative to an embankment dam are shown.
Figure 3. Cost function for an embankment dam

**Diversion dams**
As regards diversion dams a lump-sum realization cost, corresponding to 1.000.000 €, has been adopted.

**Water transfer works**
The realization costs of water transfer works shown below also include the related works as tanks and divisors.

In Figure 4 the relative function related to the realization cost of water pipes can be seen.
In Table 4 the relative function to the realization cost of tunnels is shown.

<table>
<thead>
<tr>
<th>Description</th>
<th>Quantity</th>
<th>Unitary costs (€/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excavation (m$^3$/m)</td>
<td>11,6</td>
<td>1.078,3</td>
</tr>
<tr>
<td>Concrete per cement inclusive of iron and formworks (m$^3$/m)</td>
<td>4,6</td>
<td>1.147,9</td>
</tr>
<tr>
<td>Extra charge for centering</td>
<td>2</td>
<td>360,0</td>
</tr>
<tr>
<td>others almost 10%</td>
<td></td>
<td>213,8</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td><strong>2.800</strong></td>
</tr>
</tbody>
</table>

Table 4. Cost of tunnel

In Figure 5 the functions relative to the realization costs of canals are shown.

\[
y = 123,13x^3 - 249,88x^2 + 765,51x
R^2 = 0.9948
\]
Pump stations

Total cost of pump stations depends on prevalence $H$ and on maximum water flow pumped $Q_{\text{max}}$:

$$
C = 521 \left(17.54 \cdot Q_{\text{max}} \cdot H\right)^{0.9} \left[(67 \cdot \sqrt{Q_{\text{max}} \cdot H^{3/4}})^{0.2} + 1\right] + 948(2.1 \cdot Q_{\text{max}} \cdot H + 25) + 284(17.54 \cdot Q_{\text{max}} H)^{0.8}
$$

In Figure 6, for example, curves for two values of pumped water flow are shown, from which, depending on prevalence, we obtain relative investment costs.
2.3.2. Regional Law n°19 of the 6th December 2006

The Regional Law 19/2006 of Sardinia “Disposizioni in materia di risorse idriche e bacini idrografici” (Regulations about water resources and river basins) (RAS; 2006b) accepts the National Legislative Decree 152/2006 and the European Directive 2000/60/EC.

The law regulates functions and fundamental tasks in water resources under the qualitative, quantitative and managing profile, promoting:

- responsible use and defence of water resources;
- priority supplying to civil, so to agricultural and industrial usage and the guarantee of minimal vital run-off flow;
- definition of politics in order to recover water service, resources and environmental costs.

Functions and tasks to achieve hydrogeological equilibrium are regulated in order to favour:

- prevention of water and landslide risk;
- defence and consolidation of sides of unstable areas and coastlines.

Amongst the most important regulations the following must be highlighted:

- introduction of the concept of regional multi-purpose water system (Art. 3);
- institution of “Agenzia Regionale del Distretto Idrografico della Sardegna - ARDIS” (Regional Agency of Sardinia water district) (Article 12) and of an
agency for the management of the multi-purpose water system, named
ERIS, now ENAS (Article 18);
- predisposition of the “Piano per il recupero dei costi relativi ai servizi
idrici” (Plan for recovery costs related to water service (Article 17) on which
wholesale water rates have to be defined by users; such rates have to be
unified for homogeneous categories and shall achieve what established on
WFD.

The most important articles of the Law, which support this thesis, are reported
below.

**Article 2. River basins delimitations**
1. The whole regional territory is delimited as a unique river basin under the competence of
Region and represents the river basin district of Sardinia [...].

**Article 3. Definitions**
1. According to law:

[...]

c) multi-purpose water system is the sum of works for water that, individually or as parts
of a complex system, have the possibility to feed, directly or indirectly, more territorial
areas or more different categories of users, contributing to an equalization of quantities and
costs of supply;

d) regional system of hydraulic infrastructures is the sum of works which refer to
organizations of channel, control of rivers, torrents and others bodies of water and
handmade to regulate water bodies;

e) other infrastructures are those included on water systems, aimed to single users
categories;

f) user categories are macrocategories on which water bodies uses are divided in; they are:
1) civil uses: human consumption and collective and private hygienic services;
2) agricultural uses: those related to use of water resource aimed to production of
agricultural products;
3) industrial uses: those related to water resource for industrial purposes;
4) environmental uses: those that guarantee a quote of minimal vital run-off flows
necessary for water bodies protection.

1. With the aim of guaranteeing the unity of management of the planning, programming and regulation activities on regional river basins, [...] the Regional agency of river basin of Sardinia is constituted.

2. This agency has the function of operative secretary, a functional-logistic supporting system of the Basin Authority and as a technical support for application of rules provided by the directive 2000/60/EC [...].

Article 13. Functions

1. The agency looks out of Basin Authority functions giving a technical support to work and it arranges [...]:

   a) river basin plan projects, of the relative transitional plan and the project of planning management of the river basin;

   b) an analysis of characteristics of the river basin district of Sardinia, with modes and contents provided by Article 5 of the directive n° 2000/60/EC to proceed to an examination of the impact of human activities on the status of groundwater and surface water and to define an economic analysis of water use;

   [...]  

   e) aims and objectives to produce, by the manager subject of multi-purpose water system, programs of intervention and financial plan, related to water supply service;

   [...]  

   g) regional systems of corresponding amounts for wholesale water supply for multi-purpose uses;

Article 17. Plan for recovery of costs related to water services

1. The Institutional Committee establishes yearly criteria to put into practice the principle of recovery of costs of water services depending on different sectors of use of wholesale water, on the basis of what is provided by Article 9 of directive n° 2000/60/EC. This takes account of:

   a) the need for conservation and saving of water resources to achieve objectives for a sustainable management;

   b) infrastructural investments made and to be made, which improve productivity quality and the organization of water service of regional multi-purpose water system;

   c) targets to unify criteria of economic considerations related to the water service supply of regional water system across the whole territory for categories of users;

   d) social and economic consequences of recovery of costs for different categories of users;

   e) the need to proportion on time variations of contributions for recovery of costs.
2. The agency elaborates the recovery of costs Plan related to water services for wholesale water […]; the Plan divides costs between different sectors of use and for categories of users as considered in letter f) paragraph 1 and article 3 […]

Article 18. Managing subject of regional multi purpose water system

1. The “Ente Autonomo del Flumendosa”, has been transformed into the “Ente delle risorse idriche della Sardegna”, known hereafter as ERIS (now called “ENAS - Ente Acque della Sardegna”) since the present law came into effect. It is an instrumental body of the Region that manages regional multi-purpose water system.

2. This body provides construction, management and maintenance of infrastructures, of plants and works of regional multi-purpose water system granted by the Region […].

3. Ownership of network and infrastructures and the title of licenses is property of the Region, while the management is attributed to ERIS (ENAS).

2.3.3. Studio del Modello di Gestione del Sistema Idrico Regionale - Study of Regional Water System Management Model

The Study (hereafter SMGSIR) (RAS, 2008) has the objective of analyzing regional water system and its adaptation to what is provided by the Directive 2000/60/EC and by Decree 152/2006. Its principal purpose is to determine components of financial costs of water resource both for multi-purpose system and for other sectors.

The survey involves an analysis of the present structure of Sardinian water systems showing the subdivision by sector according to uses of resources. It also describes the system of infrastructures, scheme of flows of the resource among sectors and their institutional structure, and there is an analysis of different sectors downstream the multi purpose system: civil, irrigational and industrial. Finally the average financial cost per cubic metre of multi purpose system (wholesale water cost) is calculated.

The unitary financial cost of wholesale water in 2008, calculated as a ratio between industrial cost in 2008 of the subject manager of multi purpose system ENAS, and the volumes supplied the same year which corresponds to 0,077 €/m³.

<table>
<thead>
<tr>
<th>Year</th>
<th>Industrial cost [€ x 10³]</th>
<th>Supplied volumes [m³ x 10³]</th>
<th>Unitary financial cost [€/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>46.603</td>
<td>604.485</td>
<td>0,077</td>
</tr>
</tbody>
</table>

Table 5. Unitary financial cost in 2008 for wholesale water
As stated in Chapter 9 from the regional document “unitary financial cost represents an average financial cost incorporated on resource supplied on every water system of multi purpose system and for every use”. The survey was developed by a workgroup of which the author of this thesis was a member, and represented the first important economic analysis of water service in Sardinia and represented the basis for the Management Plan of river basin District.

2.3.4. Management Plan for River Basin District of Sardinia

Management Plan for River Basin District of Sardinia (RAS; 2010), provided from Directive 2000/60/EC represents an operative instrument through which measures of defence, reclamation and improving of surface water and groundwater have to be planned and monitored. It also has to facilitate a sustainable use of water resources.

In May 2009 the first draft of the document was drawn up and has since undergone many changes. The most recent versions (March 2010) is articulated in 14 Chapters, where the following parts have been analyzed:

- description of characteristics of river basin District in reference to Article 5 of the European Directive;
- evaluation of problems linked to water resources management in reference to Article 14 of the European Directive;
- synthesis of pressures and impacts of human activities on qualitative and quantitative status of waters;
- specification of protected areas, in reference to Article 6 of the European Directive;
- synthesis of programmes of adopted measures and economic analysis on water use;
- programme of work and mode of information and active participation of the public.

Chapter 13 is dedicated to economic analysis on water use and, as indicated in the document, “it concerns informative contents provided by annex 10 part III of Legislative decree 152/2006 and from articles 5, 9 and 11 of directive 2000/60/EC”. Moreover, as stated in the same chapter, “evaluation on application of principle of water costs recovery, as established on article 9 of directive […], needs […] an estimation of prices and costs linked to water services”.

To reinforce this point, the economic analysis of the present Management Plan is based on estimation of financial costs of regional water service, reporting costs of different water sectors (multi purpose, civil, irrigational and industrial). At any
rate, there is a failure on the evaluation of environmental and resources costs and on the estimation of politics for a correct costs recovery as provided by Article 9 of the European Directive, which have to be drawn up by 22 of December 2010.

To conclude this view on reference regulations, some parts from Paragraph 13.7.2 of the present version of Management Plan are included below; here the importance and the necessity of usage of the mathematical model of hydro-economic optimization for water resources systems to achieve the instructions of WFD is emphasized.

“Resources Costs” […] can be evaluated through optimization models.

Resources Cost is different in time and space depending on water availability […] and on dynamics of water demand and on “willingness to pay”. Such variable can be captured only through optimization and simulation of hydro-economics models, so their development is essential for a correct determination of resources costs.

These models are very useful for the analysis of environmental variables which contribute to the environmental status of water bodies, as a minimal vital run-off of water bodies and the minimal level on reservoirs.

Use of hydro-economic models is necessary to analyze such problems on basin scale, reproducing the relationship between surface waters and groundwater, the infrastructural system complexity, the operative rules which manage it and the demand functions for different users.

Mathematical models used for the Management Plan have been utilized for this thesis and have represented an operative instrument to achieve results that shall be presented later on.
3. Costs allocation problem

How should the common costs of an enterprise be shared “fairly” among its beneficiaries? This problem is widespread, both in public enterprise and also within private firms. It arises in the pricing policies of public utilities providing telephone services, electricity, water, and transport. It occurs in the cost-benefit analyses of public works project designed to serve different constituencies, such as a multipurpose reservoir. It is implicit in the determination of access fees or user charges for common facilities such as an airport or waterways. In private corporation it occurs in the form of internal accounting schemes to allocate common and overhead costs among different divisions of the firm (Young, 1985).

Such cost allocation problems typically exhibit two features:

- cost must be allocated exactly, with no profit or deficit;
- there in no objective basis at hand for attributing costs directly to specific products or services.

3.1. Cost allocation methods

Common practice principal cost allocation methods are the following:

- **Egalitarian methods**: equal allocation of costs between persons. It results the easiest way which is usually used in cases where persons have homogeneous characteristics.

- **Proportional methods**: subdivision of total cost proportioned to a determined characteristic measure as for example the quantity of good used.
- **Methods based on marginal cost.** Economic theory suggests that optimal rate is the one which maximizes consumer’s surplus, i.e. the rate which guarantees the maximum covering of a demand for a linear rate equal to marginal cost. Methods based on marginal costs are applicable in case total covering commitment of costs is not enforced.

- **Methods based on separable costs.** The component of total cost strictly related to a person is directly assigned to them and the remaining part is allocated through others methods, for example through the proportional one.

- **Arbitrary methods or opportunity methods:** these are methods based on imposed rates which do not derive from a particular methodology; in fact, they are the result of valuation and calculation on the basis of an experience developed on the specific sector.

Other criteria of cost allocation among users are present, and they are based on the usage of specific mathematic formulations more or less complex which are corresponding to specific property, according to requests made by promoting enterprise.

### 3.1.1. Fee methods in water resources

In water resources, the most used methods are the proportional ones, where there is an assignment of costs in function of the amount of resource used or, in case of irrigational demands, in function of the number of irrigated hectares.

For years in Italy assignment of subventions, subsides, refunds and grants-in-aid provided by the central government to companies, syndicates and operators of water system, has been, and is still a common practice. They are almost obligatory. Such practice led to use criteria of assignment which do not guarantee total coverage of sector’s costs.

Moreover, only for the drinking sector, a rates regulation method has been defined, the so called “metodo normalizzato” (normalized method) established by Law 36/1994 and now used in every national ATO. This method was based on revenue cap criterion, according to which operator’s proceeds are established for one year and, at the same time, maximum limits to growing of reference rate which are settled, are calculated under the following formulation:

\[
T_n = (C + A + R)_{n-1} \cdot (1 + P + K)
\]

where:

- \( T_n \) is the rate for the current year;
- \( C \) is the component of operative costs;
A is the component of amortization’s costs;

R is the component which indicates remuneration of investment;

P is the inflation rate programmed for the present year;

K is the “limit price”, maximum growth rate for rate beyond P.

Once determined reference rate every AATO shall determine real rate which will be applied to final users.

For years normalized method has had many blames and few occasions of application, so water service operators, after having mentioned many times conflicting aspects, proposed a better overcoming. Many changes have been put forward even though such method is still in force on its original formulation.

As regards other water sectors, at present costs allocation methods used follow closely those seen before. Nevertheless, such methods present a big arbitrariness of usage in all national territory.

In Sardinia, different criteria have been carried out, depending on users’ typology and usage level of water resource.

In regard to the regional multipurpose system, the rates applied to the three macro users (civil, irrigational and industrial) are proposed yearly by ARDIS and approved by the “Comitato Istituzionale Regionale” (Regional Institutional Committee) which belongs to Basin Authority. Their determination looks like a product of estimates and evaluations free from any criteria of calculation; and it is, in fact, totally absent a methodology which justify their application.

The following rates indicate the period from 2005 to 2010.

<table>
<thead>
<tr>
<th>ENAS rates [€/mc]</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil I</td>
<td>0,1</td>
<td>0,07</td>
<td>0,056</td>
<td>0,056</td>
<td>0,025</td>
<td>0,025</td>
</tr>
<tr>
<td>Civil II</td>
<td>-</td>
<td>0,1</td>
<td>0,07</td>
<td>0,07</td>
<td>0,056</td>
<td>0,056</td>
</tr>
<tr>
<td>Irrigational I</td>
<td>0,02</td>
<td>0,02</td>
<td>0,015</td>
<td>0,007</td>
<td>0,007</td>
<td>0,005</td>
</tr>
<tr>
<td>Irrigational II</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0,015</td>
<td>0,015</td>
<td>0,015</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,23</td>
<td>0,23</td>
<td>0,23</td>
<td>0,23</td>
<td>0,23</td>
<td>0,23</td>
</tr>
</tbody>
</table>

Table 6. ENAS rates 2006 – 2010 – Source: Management Plan

Starting from 2006 for the civil demand and from 2008 for irrigational demand, two rates have been applied depending on the amount of used water resource. For the civil demand the first rate is adopted for a maximum consumption of 130 millions of cubic meters and, after this, the second rate has to be applied. Instead, as it regards to irrigational demand, basic volumes in function of the amount of the resource consumed the previous year are yearly defined for every land
reclamation syndicate. If such syndicate is able to consume a volume of resource within the basic volume assigned it will be adopted the first rate, on the contrary it will be applied the second rate to the exceeding resource.

From the analysis of data it is clear that industrial demand is the only one which does not exhibits changes in rates. However, the civil and irrigational demand registered a reduction of the 75% during the last 6 years. As we said, this trend is decided without any criterion.

Moreover, income for rates for multi purpose system does not guarantee the total recovery of costs of operator ENAS. Within 2010 they can cover almost the half (Table 7).

<table>
<thead>
<tr>
<th></th>
<th>[M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure Budget ENAS 2010</td>
<td>39,980</td>
</tr>
<tr>
<td>Income Budget ENAS coming from rates</td>
<td>17,624</td>
</tr>
</tbody>
</table>

**Table 7. Budget plan ENAS 2010. Source: Management Plan**

In the case of irrigational sector rates applied by Sardinian Land-Reclamation Syndicates are different. Such operators recover financial costs of water services through two sources:
transfers from Central Government and from the Region;
- rate applied to their associated users.

Generally, at the end of every year they quantify final costs and, depending on the
amount of received grants-in-aid, establish rates for a break even. On the basis of
such method, due to variability of year costs and of income of contributions, there
is a big variation of applied rates among these operators, and also inside the same
between one year and the following one.

Several times a yearly fixed rent plus a variable fixed rent in function of irrigated
hectares or consumption of resources is assigned to irrigational final user. But
every syndicate fixes rates as he wants. Also, in this case it is very arbitrary when
it comes to define the associated costs to final users due to a failure of a unique
allocation costs methods. Moreover, for their intrinsic definition due to public
grants, rates do not guarantee recovery of costs of irrigational water service.

In such a context the Management Plan reports that Land reclamation Syndicates
will have to modify their methods of rating, trying to unify to an unique criterion
acceptable for all the operators.

As far as the industrial sector is concerned, three situations can be settled:
- areas linked to municipal water system. In this case a rate established by
  AATO is applied;
- users linked to multipurpose water system by cooperative aqueducts. They
  pay rate fixes by ENAS;
- users which dispose of concessions for water use. In such case an annual tax
  is in force.

3.2. Willingness to pay

One of the worst problems of cost allocation methods that are found, is that they
almost completely ignore the problem of motivation: “why, for example, should
agents accept an allocation that exceeds their opportunity costs (i.e. the cost
related to the next-best choice available to someone who has picked among several
mutually exclusive choices) or willingness to pay?” (Young, 1985).

To better explain the concept of “willingness to pay” here it is presented a simple
example which follows the one proposed by Young (1994).

Two nearby cities [A] and [B], 50 thousand residents the first and 10 thousand the
second one, have to build a water distribution system. If the two cities would
decide to build separately a facility for itself, [A] afforded a cost of 20M€, while [B]
10M€. If they cooperated, id est in case of realization of a unique facility serving
both communities, total cost would be of 25M€, which would be lower respect to
the sum of single costs of the two autonomous aqueducts (30M€). Clearly it makes
sense the second solution, since they can jointly save 5M€. Cooperation will be
possible if the two users agree on how to divide the charges for project’s realization.

<table>
<thead>
<tr>
<th>Residents</th>
<th>Cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>City A</td>
<td>50,000</td>
</tr>
<tr>
<td>City B</td>
<td>10,000</td>
</tr>
<tr>
<td>City A+B</td>
<td>60,000</td>
</tr>
</tbody>
</table>

Table 8. Example of the two cities

One possible solution to costs sharing would be to divide total cost in equal parts between the two cities, according to an equalitarian method, which means 12,5M€ for each. Such division means the same power for both cities and this would be acceptable if the two cities have the same dimension. Otherwise, in such case, the same division would imply that every resident of the city [A] paid only one fifth respect to what paid [B], even though they use the same facility. This hardly seems fair and it is predictable that [B] will not participate to this cooperation project.

Another possible solution would be to divide the costs equally among residents, according to a proportional method, obtaining a per capita cost of 416,67€. So [A] would have a total cost of 20,8M€ while [B] of 4,2M€.

<table>
<thead>
<tr>
<th>Town A</th>
<th>Town B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of building their own aqueduct</td>
<td>20 M€</td>
</tr>
<tr>
<td>Equal division of costs between cities</td>
<td>12,5 M€</td>
</tr>
<tr>
<td>Equal division of costs among persons</td>
<td>20,8 M€</td>
</tr>
</tbody>
</table>

Table 9. Costs allocation for the two cities (1)

Such proposals do not take into account the opportunity costs of the parties participating on the project. In fact, on the first case [B] is not likely to agree since it would afford a bigger cost respect to what would afford for an own aqueduct, similarly [A] is not like to agree to an equal division per capita.

Here three more criteria of cost allocation will be proposed. They take into account availability of paying from the two cities.

The first criterion proposes to divide in equally saving due to cooperation of the two cities. Saving corresponds to the difference between the cost of project of the two autonomous aqueducts and the cost of common project, i.e. 5M€. If we divide saving in equal parts and we subtract it to the opportunity cost we obtain:

- for [A]: 20M€ - 2,5M€ = 17,5 M€;
- for [B]: 10M€ - 2,5M€ = 7,5M€.
A second criterion, otherwise, is to divide saving in equal parts among the residents; here it corresponds to a per capita saving equal to 5M€/60000ab = 83,3 €/residents, which would determine:

- for [A]: savings of 4,17M€, associated cost 20M€ - 4,17M€ = 15,83M€;
- for [B]: savings of 0,83M€, associated cost 10M€ - 0,83M€ = 9,17M€.

A third solution would be to subtract savings in proportion to opportunity cost of the two users.

- for [A] savings (5*20/30) M€ = 3,3M€, associated cost 20M€ - 3,3M€ = 16,7M€;
- for [B] savings (5*10/30) M€ = 1,7 M€, associated cost 10M€ - 1,7M€ = 8,3M€.

Note that this is the same thing as allocating total cost in proportion to each city’s opportunity cost:

- for A: (25 * 20/30) M€ = 16,7M€;
- for B: (25 * 10/30) M€ = 8,3M€.

<table>
<thead>
<tr>
<th></th>
<th>Town A</th>
<th>Town B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of building their own aqueduct</td>
<td>20,0 M€</td>
<td>10,0 M€</td>
</tr>
<tr>
<td>Equal division of savings between cities</td>
<td>17,5 M€</td>
<td>7,5 M€</td>
</tr>
<tr>
<td>Equal division of savings among persons</td>
<td>15,83 M€</td>
<td>9,17 M€</td>
</tr>
<tr>
<td>Savings proportional to opportunity cost</td>
<td>16,7 M€</td>
<td>8,3 M€</td>
</tr>
</tbody>
</table>

Table 10. Costs allocation for the two cities (2)

Such example shows several aspects. First, there doesn't exist an easy unique answer for costs allocation, even if it apparently looks simple. In fact, it is impossible to find just one criterion to follow; it is necessary to analyze in details every case.

Moreover, it is important to underline the absence of market mechanisms to value a solution. Actually, one solution would be to imitate market behaviour equalizing charges to marginal costs, equal to the difference between the cost with/without the relative city. In this example we obtain:

- for [A] 25M€ - 10M€ = 15M€;

However, in this case, the sum of marginal costs do not correspond to total costs, i.e. such sum doesn’t cover all the charges; so it is not an acceptable criterion if covering costs is considered a basic principle to guarantee.

Moreover, dealing with costs allocation, it is necessary to consider some concepts interconnected among themselves: efficiency, equity and sustainability. Such instruments are basic to reach a solution of cooperation among the players in
order to achieve better savings and to obtain the most economically efficient solution.

### 3.3. Fair allocation and Cooperative Game Theory

As we could observe, a fair cost allocation has to comply with general principle of equity, efficiency, and justice. For that it shall respect principles of individual acceptance and agreement among users in order to support their voluntary cooperation.

Moreover the majority of cost allocation methods have the default of not underlining motivation on criteria in assigning water services costs. So, one of the main objectives of this study is the research for a shared cost allocation, giving an adequate justification of the adopted criteria.

The main problem is not in finding a modality of cost allocation among users, i.e. searching a determined law of charges allocation, but rather how to allocate equally and fairly sustained costs. This means to find an impartial allocation of costs for all the users of such project, in order to promote and guarantee cooperation among users and the feasibility of a common project which can allow reduction of costs for all the beneficiaries.

The goal of analysis is to devise criteria and methods for solving these problems in a just, equitable, fair and reasonable manner. Cost allocation is thus ultimately concerned with fairness. The methods and principles of cost allocation that are likely to find acceptance must somehow be grounded in primitive, common-sense ideas of fairness and equity (Young, 1994).

But precisely what is meant by the word fair? According to Webster (1981), it stems from *fagār*, an Old High German term meaning "beautiful". *Fair* means, firstly, "attractive in appearance: pleasant to view". Significantly, a secondary meaning is "pleasing to hear: inspiring hope or confidence often delusively ... specious". It is closely connected to such ideas as *just, equitable, impartial, unbiased, objective*. "Fair ... implies a disposition in a person or group to achieve a fitting and right balance of claims or considerations that is free from undue favouritism even to oneself ... Just stresses, more than fair, a disposition to conform with, or conformity with the standard of what is right, true, or lawful, despite strong, especially personal, influences tending to subvert that conformity" (Young, 1994).

On the basis of such considerations, the definition of a criterion of equitable and impartial costs assignment represents the basis to guarantee binding agreements among interested users, by supporting cooperation to achieve more efficient solutions and by ensuring stability of consensual solutions. On the contrary, the impossibility to define an unique criterion of costs assignment, which can result satisfactory for all the parts, can provoke total or partial abandonment of
cooperative projects, so causing a loss of efficiency and an increase of externalities produced by individual projects. (Deidda, 2009).

As reported by Young (1994) Cooperative Game Theory provides necessary instruments to analyze situations in which it is basic to research a sharing mechanism considered efficient, fair and it has to supply appropriate incentives among the parts. Lemaire (1984) points out that a cost allocation problem is identical to the determination of value of a cooperative game with transferable utilities. Sharing cost between users can be seen as a kind of game where it is necessary to determine the fair allocation among different players. A cooperative game with transferable utilities belongs to the science called Game Theory.

By using techniques of assignment of costs belonging to Game Theory, it suggests to possibly make the process of negotiation explicit through mathematic formulas which focus on such properties that guarantee equity, justice and cooperation among users involved in a project, with the objective to obtain an acceptable solution for everyone.

Using such procedures, cost allocation results an inside procedure to the project because there is an a priori solution. Such denomination is in contrast with what usually occurs when costs allocation is an external procedure to planning steps and it is considered only once such project is already carried out, i.e. an a posteriori solution.

Criterion of costs allocation does not have to be a support but rather the result of a decision making process.
4. Game Theory

Game Theory (hereafter referred to as GT) is a mathematical science developed around the first half of the last century whose origin was to solve situations of conflict among different players in conditions of cooperation or competition. The name comes from “Theory of Games and Economic Behavior” written by Von Neumann and Morgenstern (1944).

According to Parrachino et al. (2002) GT is the study of mathematical modelling of strategic behaviour of decision makers (players), in situations where one player’s decisions may affect the other players. Contrarily, in respect to classic Operative Research on which decisions relative to a problem are taken by one player who can play in complete autonomy and freedom, GT deals with situations on which the result depends on choices made by different people, called players. These participants operate to achieve objectives that can be in common, different and even in contrast (Fragnelli, 2010). The basic assumption is that decision makers are rational players, “intelligent”, so they take into account other decision-makers’ behaviours.

GT is quite recent and conventionally begins in 1913 thanks to Zermelo who analysed different strategies for chess. Such game belongs to qualitative games where the only objective is to win, the sum is equal to zero (if you win, the other loses: eat or be eaten) and you know all the information (you play fear). Until the fifties, GT dealt with similar cases, of limited importance.

The firsts step was in the forties when in 1944 the book written by Von Neumann e Morgenstern was published. They were respectively a mathematician and an economist who introduced the concept of “strategy” and proposed different applications. Strategy is commonly defined as “policy” that individuates in every game’s situation a “move” among the many possible ones.
Another incentive was given by the mathematician John Nash, who, in 1950, introduced the concept of *Nash equilibrium* and in 1953 that of *bargaining equilibrium* which is the “typical” solution of a “non-cooperative game”. The importance of Nash's discoveries was treated on the famous film “A Beautiful Mind”.

During the same period Shapley (1953) indicated another solution to solve “cooperative” games, i.e., those games on which the players can make binding agreements among themselves to improve their condition.

Subsequently Harsanyi (1968) expanded the research to incomplete information games, i.e., those games on which players do not know all the characteristics of the game, as, for example, the reward for the other participants.

Currently, GT plays a very important role in economy. In fact many publications have been produced for such theory and, thanks to studies relative to GT, scientists like Nash, Harsanyi and Selten in 1994, Mirrlees and Vickrey in 1995, Schelling and Aumann in 2005 and Hurwicz, Maskin and Myerson in 2007 received the Nobel prize.

Besides economic world, the applications of GT ranges from military to politic sector, from psychology to informatics, from biology to sociology and also concern studies related to sport (Camerer, 2000; Hofbauer & Sigmund, 2003). For instance, equal sharing of properties and heritage is an area which can be studied through TG techniques (Aumann, 2006); in social science we can find interesting applications regarding the study of power distribution in legislative procedures (Brams, 1975; Odershook, 1986); sociologists developed a whole branch of the theory in order to study group’s decisions matters (Parson & Wooldridge, 2000); even the epidemiologists use the GT, specially for immunization procedures and methods to verify vaccines and other medicines (Roth, 1984); finally a very used sector of application is military strategy (Aumann, 2006).

If we enter into details of the structure of GT, we can distinguish two branches of games, according to the classification made by Harsanyi (1966):

- non-cooperative games, where binding agreements among the players are not possible;
- cooperative games, where binding agreements among the players are possible.

The main distinction between the two games is that the first games analyze situations where the players consider only their own strategic objectives and thus binding agreements among the players are not possible, while the second ones are mainly based on agreements to allocate cooperative gains among the players (Parrachino et al., 2002).

Cooperative games can be divided, in their turn, in:

- non transfer utility-games, where the players receive a pre-assigned payoff (the value of the utility or the payment);
- transfer utility-games where the players of a coalition can divide the utility, that they have, in any possible way.

Depending on the models of game, even the searched solution takes different meanings: in the case of non-cooperative game or non transfer utility-game the solution of the game consists in giving indications to one or more players, possibly all the players, about strategies to adopt; but, for transfer utility-games it means to determine a division of the winning among all the participants.

4.1. Non-cooperative Game Theory

In this branch of games, binding agreements and communication among the players are not possible, apart from the fact that their objectives are in contrast or common and that they can have interests in coming on agreements (Fragnelli, 2010). So every player plays in an autonomic way in function of possible actions which other participants of the game shall fulfil.

Non-cooperative Game Theory (NCGT) allows to interpret paradoxical phenomena on which it is possible to show how the research of an optimal solution for every player can lead a general loss of well-being, contrary to what stated by the theory of efficiency according to Pareto. In order to explain such concept, here below it is presented one of the most classic examples of non-cooperative game.

Prisoner’s dilemma

The so-called “prisoner’s dilemma” is, probably, the most famous problem of NCGT, introduced in 1958 by Dresher and Flood (Flood, 1958) and previously adapted in an informal way by Tucker in 1953 (Tucker, 1953).

Two persons A and B are arrested by the police for the same crime and are interrogated separately by the judge. Every person can choose, independently to confess [C] or not to confess [NC].

If both of them do not confess, they will be condemned for small punishment with 2 years of prison, but if they both confess they will receive a five years sentence; if one confesses and the other not, the one who admits obtains a reduction of sentence and he gets a one year sentence while the other gets an aggravating circumstance and he is condemned for six years. The punishments are reported on the following table.
Table 11. Game of the prisoner

<table>
<thead>
<tr>
<th></th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[C]</td>
</tr>
<tr>
<td>A [C]</td>
<td>5; 5</td>
</tr>
<tr>
<td>A [NC]</td>
<td>6; 1</td>
</tr>
</tbody>
</table>

Rationally A chooses [C] because he can get a small punishment whatever the choice of B is (5 < 6; 1 < 2) and similarly also B chooses C. So the strategy of not confessing is dominated by the confessing strategy. By eliminating dominated strategies we can get to Nash equilibrium (1950) where the prisoners both confess and are condemned to a 5 years sentence. The expected decision is [C, C], while more advantageous for both the prisoners would be [NC, NC] and the consequent two years of prison for every one (Pareto optimum).

If we consider that the two players had previously decided the common choice of not to confess in case of arrest, once closed in two separate cells every prisoner would wonder if the other prisoner’s promise is going to be maintained. Every one will have the dilemma if to confess or not to confess. TGNC shows that it exists only one point of equilibrium [C, C].

Prisoner’s dilemma was used to describe the situation between USA and USSR during the Cold War. Considering the two superpowers as the two prisoners, the choice to confess as the atomic armament and, on the other hand, the choice not to confess as unilateral disarming, it is clear that in that period it was necessary the arms race for the two countries, even if such final result was not optimal for anyone of the two superpowers and neither for the whole planet.

To conclude, it is suitable to reflect on the concept of Nash equilibrium. This is the product of the dominating strategy of every player; it represents, therefore, the situation of the game in case in which every player realizes what is the best for himself, trying to maximise his own profit apart from the other participant’s choice. However, as we said, it doesn't mean that Nash equilibrium is the best solution from a general point of view, so it is possible that a group of players or, at least, all the participants of the game can improve their situation going away from the equilibrium. In fact, as we said, Nash equilibrium cannot be a Pareto optimum, so, there can be other combinations which conduct to improve the gain of some players without reducing the game of others; in extreme case, it is possible to improve the situation of all the participants of the game.
4.2. Cooperative Game Theory

Cooperative Game Theory (CGT), as the name reports, differently from NCGT, analyses situations in which participants can cooperate among themselves in order to achieve a common purpose. The players are not obliged to have contrasting interests, so it is possible that some of them tend to cooperate to improve their result. In order to cooperate, it has to be possible, above all, to communicate and to make agreements among the players. Furthermore there must be an authority necessarily strong and accepted by all the participants able to make such agreements respected (Fragnelli, 2010). In particular, CGT tries to supply answers about how to share gains and costs of a common action among participants in order to guarantee those principles of equity, individual acceptability and general agreement among the players.

As we said before, cooperative games can be divided in:

- non transfer utility-games, where the players receive a pre-assigned payoff (the value of the utility or the payment);
- transfer utility-games where the players of a coalition can divide the utility in any possible way.

Such research belongs to the second branch for which three hypotheses must be satisfied (Fragnelli, 2010):

- from a normative perspective, the transfer of utility among the players must be possible;
- there must be a common mean of exchange, for example money, through which it is possible to transfer utility from a material perspective;
- utility functions among the players must be equivalent, for example linear functions of the quantity of money.

Now it is necessary to explain in details the most important elements on the basis of CGT with transferable utility. Afterwards the most used concepts, principles and solutions will be illustrated; so, at the end of the chapter, some applications of CGT in literature will be presented.

4.2.1. CGT with transferable utility

Firstly the following basic elements can be defined (Young, 1994):

We will refer $N = (1, 2, ..., n)$ as a set of players participating in the game. Every subset $S \subseteq N$ is called “coalition”, and for $S = N$ we have the “Grand Coalition”. The players can represent real subjects, as the users of a water system, or member of a more abstract set as the sector of a company, or they can also represent different planning alternatives to realize commonly or separately.
\( c(i) \) represents the cost connected to the user \( i \) considered in a independent and autonomous way with respect to the other players, called “stand-alone cost”, and \( c(S) \) the cost linked to coalition \( S \), i.e. the cost commonly sustained by all the users of \( S \). Consequently, \( c(N) \) is the cost associated to the Grand Coalition, i.e. the common cost sustained by all the participants of the game. Finally by convention the cost linked to an empty coalition corresponds to zero \( c(\emptyset) = 0 \).

The cost associated to a generic coalition (formed by only one player, by their sets or by all the participants of the game) represents the least cost of serving such coalition by the most efficient way, i.e. the minimum cost necessary to satisfy its players. Discrete function \( c \) formed by the set of all associated costs to all the possible coalitions is called “characteristic function” of a cooperative game.

Here it is presented a simple example of cooperative game to explain in a clear way mostly above-stated concepts.

We consider the realization of a multipurpose reservoir necessary to satisfy three different objectives: regulation, flood control and hydroelectric production; every one of them represent a different player. The realization of a dam represents a case of project characterized by economies of scale for which cooperation among different users is fundamental to obtain economic savings. In fact, it is more convenient to realize just a unique large-sized work which satisfies the different users, instead of the construction of more works for every user.

In function of the number and the type of user there is a variation on height of the work and consequently the relative cost of construction. We hypothesize the following costs:

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regulation (R)</td>
<td>160</td>
</tr>
<tr>
<td>Flood control(F)</td>
<td>140</td>
</tr>
<tr>
<td>Hydroelectric (H)</td>
<td>250</td>
</tr>
<tr>
<td>R + F</td>
<td>300</td>
</tr>
<tr>
<td>R + H</td>
<td>380</td>
</tr>
<tr>
<td>F + H</td>
<td>370</td>
</tr>
<tr>
<td>R + F + H</td>
<td>410</td>
</tr>
</tbody>
</table>

Table 12. Characteristic function

So the different combinations of players are considered in order to examine all the possible planning alternatives and to value the most convenient (Table 13.).
As we can see the Grand Coalition is the most efficient solution from an economical perspective for the system. Consequently, the problem which arises now is how to share the cost of the Grand Coalition in a fair and acceptable way for all the participants of the game. By using allocation methods of CGT it is possible to answer such question.

### 4.2.1.1. Definitions

a) If for every pair of disjoint coalitions $S'$ and $S''$ we have:

$$c(S' \cup S'') \leq c(S') + c(S'')$$

then the characteristic function $c$ is called subadditive and the relative game “subadditive”.

If a game is subadditive then the players are stimulated to cooperate, because the unions of the two group of players will determine a cost lower than the sum of the autonomous costs. A game with a characteristic subadditive function will be characterized by economies of scale, so the Grand Coalition will be the most efficient from an economical perspective. This is the case when it is economically more convenient to realize common projects rather than independent projects.

b) If for every pair of coalitions $S'$ and $S''$, such as $S' \subseteq S''$, we have:

$$c(S') \leq c(S'')$$

then the characteristic function $c$ is called “monotonic”.

It represents the situation in which the cost of a determined project increases as the number of participants to the same project increases.
4.2.1.2. Principles

In order to determine a costs sharing in line with the criteria of efficiency, equity, acceptance and incentive for cooperation it is necessary to consider the following principles.

**Rationality principle**

In order to stimulate the cooperation among the players to achieve commonly a determined project, it must be guaranteed the principle defined by Ransmeier (1942) as “stand alone come test”. Such principle can be extended to every individual player; for this reason it is called *individual rationality*:

\[
x_i \leq c(i)
\]

where \(x_i\) is the quantity of cost assigned to a player; or *group rationality* when it is referred to single coalitions.

\[
\sum_{i \in S} x_i \leq c(S)
\]

According to such principle, no player or group of players, forming a coalition, would accept a cost assignment lower than the cost which he/they would sustain participating in an autonomous way, i.e. upper than its own opportunity cost.

**Marginality principle**

The other principle is the so-called marginality principle or “*incremental cost test*” (Young, 1985).

In general incremental cost or marginal cost of a coalition \(S\) is defined as the quantity

\[
c(N) - c(N/S)
\]

According to *marginality principle*, the following condition must be verified:

\[
\sum_{i \in S} x_i \geq c(N) - c(N/S) \quad \forall \ S \subseteq N
\]

Every player or set of players will have to sustain at least his own marginal cost of entry in a coalition. Otherwise, the coalition of pre-existent players will be in an inefficient condition to finance the entry of the new player or set of players into it.
Rationality principle produces an incentive to the voluntary cooperation among the players, while the principle of marginality supplies conditions of equity in the game (Young, 1994).

4.2.1.3. Game solutions

As we said before, CGT with transferable utility tries to give answers about the modalities of costs (or benefits) sharing of a common action among the participants. Therefore the solution of a cooperative game means to define a useful criterion to share the costs commonly sustained by the players in an efficient and fair way. Such criterion has to supply adequate incentives for the cooperation among the players.

In details, CGT supports solutions which include all the participants of the game, so the majority of solving methods refer to the cost sharing of the Grand Coalition (Parrachino et al., 2002). Under such hypothesis, a generic solution conforms with sharing defined by a vector \( x \) of components \( [x_1, x_2, \ldots, x_n] \) such as

\[
\sum_{i \in N} x_i = c(N)
\]  

(10)

where \( x_i \) is the quantity of cost assigned to player \( i \).

The previous equation satisfies the principle of efficiency, according to which the cost of the Grand Coalition is totally divided among all the participants of the game; under equation (10) it is shown that the principle of marginality and rationality are equivalent.

The solution of a transferable utility game can be grouped in two branches (Fragnelli, 2010):

- set-theoretical solutions which individuate a set of vectors which share the value of the game among all the players
- point solutions which individuate only one division and are more similar to the classic idea of solution of a problem.

4.2.1.3.1. Set-theoretical solutions

Core of a cooperative game

The core of a cooperative game, defined by Gillies in 1953, is the set of allocations \( x \in R^N \) such that those conditions expressed from the (7) and (10) equations, or at the same way (9) or (10), have to result valid for every \( S \subseteq N \). The core is a closed, compact, convex subset of \( R^N \), but unfortunately it can results empty.
The condition which guarantee the existence of a core is that the characteristic function must be concave (Shapley, 1971), so:

\[ c(S \cup T) + c(S \cap T) \leq c(S) + c(T) \quad \forall \; S, T \subseteq N \]  

(11).

The core of a cooperative game represents a subset solution; inside there are several possibilities of cost allocation which respect efficiency, equity and incentive principles to the cooperation. These divisions of cost inside the core are such that no player can improve his own condition without making the others players condition worse.

Let’s consider, for instance, the game of multi purpose reservoir described on paragraph 4.2.1. Firstly it is necessary to apply opportunely principles of rationality and marginality through which the values of maximum and minimum cost attributable to the three players can be defined. In fact, through the principle of individual rationality, see equation (6), the maximum cost sustainable by the player is determined, while the minimum cost is supplied from the principle of marginality for every single player (or in an equivalent way from the principle of rationality associated to coalitions), as expressed by equation (9). The results are expressed on Table 14.

<table>
<thead>
<tr>
<th>Rationality</th>
<th>Marginality</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(R) ≤ 160</td>
<td>C(R) ≥ C(N)-C(L+I) = 40</td>
</tr>
<tr>
<td>C(L) ≤ 140</td>
<td>C(L) ≥ C(N)-C(R+I) = 30</td>
</tr>
<tr>
<td>C(I) ≤ 250</td>
<td>C(I) ≥ C(N)-C(R+L) = 110</td>
</tr>
</tbody>
</table>

Table 14. Maximum and minimum values of cost

Consequently, the core of the game can be analytically represented in the following system:

\[
\begin{align*}
\text{Regulation} + \text{Flood control} + \text{Hydroeletrical} &= 410 \\
40 &\leq \text{Regulation} \leq 160 \\
30 &\leq \text{Flood control} \leq 140 \\
110 &\leq \text{Hydroeletrical} \leq 250
\end{align*}
\]  

(12).

Furthermore, it is possible to represent graphically the core through a triangular diagram, as shown in Figure 8.
The triangle is equilateral, its heights are proportional to the cost of the Grand Coalition and its points represents the set of possible (positive) allocations of cost. Every side is representative of a player and the distance between one of them and one point inside the triangle gives us the allocation cost related to the player. For instance, the barycentre gives us an equalitarian sharing of cost while the vertices correspond to the situation in which total cost is completely assigned to one user. The dashed lines represent the maximum and minimum cost sustainable by every player according to inequalities in (12) and, consequently, the painted area represents the core of the game.

If the number of players is $n > 3$ the core can be represented in a space with a $n-1$ dimension.

**Games with an empty Core**

Sometimes the core of a cooperative game can results empty if the characteristic function is not concave. If we consider, for example, the following three-players game, wherein:

\[
\begin{align*}
    c(1) &= c(2) = c(3) = 6 \\
    c(1,2) &= c(1,3) = c(2,3) = 7 \\
    c(N) &= 11
\end{align*}
\]
In such case, even if the characteristic function is subadditive, which means that the Grand Coalitions is the most efficient solution from an economic point of view, the game results with an empty core. In fact if we applying the marginality and rationality principles we obtain for every player:

\[ 4 \leq c(i) \leq 6 \]

and utilizing the triangular diagram we can see the emptiness of the core, as shown in Figure 9.

![Figure 9. Game with an empty core](image)

So it involves that every allocation which share the cost of Grand Coalition among the players will be unstable, because at least one coalition will be stimulated to leave the group and to “play” autonomous.

In fact, in the example the principles of rationality and marginality supply the same results for all three players because they have the same importance in the game. Therefore an equitable allocation could be the one that shares equally the cost of the most efficient alternative among the three players:

\[
x_1 = x_2 = x_3 = \frac{c(N)}{n} = \frac{11}{3} \approx 3.67
\]

But such division can not be convenient for two players who decide to cooperate excluding the third player. In fact, if the players 1 and 2 formed a coalition and divided their own cost equally, we would obtain:
\[ x_1 = x_2 = \frac{c(1,2)}{n - 1} = \frac{7}{2} = 3.5 \]

so there should be a saving respect to the previous allocation. Such situation will
not be favourable for the third player who will be obliged to sustain a higher cost
of \( c(3) = 6 \) and, above all, such a situation won’t be favourable for the whole
system because the total cost will be

\[ c(1,2) + c(3) = 7 + 6 = 13 \]

which do not represent the most efficient solution from an economic perspective.

We deduce from this simple example that it is necessary that the core won’t be
empty in order to obtain an equitable and fair sharing which stimulates the
cooperation among the users and the research of the most efficient solution.

If the core is empty the cooperation among the players is not spontaneous but it
can be forced by an external authority modifying the conditions of the game in
order to allow the creation of the core, for example, by imposing a tax for every
subcoalition of \( N \).

The imposition of a tax is a modality to stimulate the players to cooperate among
themselves, i.e. an incentive to cooperation in order to achieve the most efficient
solution for the whole system.

**Least core**

To determine the obtainable least core we have to value, through linear
programming, the minimum tax \( \delta \) to impose, i.e. in mathematical terms (Einy et al.,
1998):

\[
\begin{align*}
\min \delta \\
\text{s.t.} \sum_{i \in S} x_i &\leq c(S) + \delta \quad \forall S \subset N \\
\sum_{i \in N} x_i &= c(N)
\end{align*}
\]

**Proportional least core**

It is a variation of the previous model which considers a proportional tax to the
cost of every coalition. To obtain this condition, the linear programming problem
to be solved will be:
\[
\begin{align*}
\min & \alpha \\
\text{s.t.} & \sum_{i \in S} x_i \leq c(S)(1 + \alpha) \quad \forall S \subseteq N \\
& \sum_{i \in N} x_i = c(N)
\end{align*}
\]

(14).

4.2.1.3.2. Point solution concepts

As we have seen the core of a cooperative game represents a set of acceptable solutions. In many cases, however, it is a good idea to supply a unique cost allocation among the players, in order to define only one vector which satisfies the equation (10). So we have the problem to choose among different possibilities.

The principal and most used point solution concepts of CGT are:

- the Alternative Cost Avoided method (ACA);
- Shapley value;
- the nucleolus and its variant “per-capita”.

Alternative Cost Avoided method (ACA)

Given a cooperative game for an established player \( i \) we can define:

- separable cost: its marginal cost as regards the Grand Coalition:
  \[ m_i = c(N) - c(N - i) \]
- alternative cost: its stand-alone cost: \( c(i) \).
- remaining benefit: the difference between the alternative cost and the separable cost: \( r_i = c(i) - m_i \).

Such ACA method assigns the cost to many players according to the following formula:

\[
x_i = m_i + \frac{r_i}{\sum_{j \in N} r_j} \left[ c(N) - \sum_{j \in N} m_j \right]
\]

(15).

According to it, every player pays entirely his own separable cost, while non separable costs \( [c(N) - \sum_{j \in N} m_j] \) are divided in proportion to the remaining benefit.

This method is valid if \( r_i \geq 0 \); this is possible if characteristic function is subadditive. Furthermore for a maximum three-player game, with a subadditive
characteristic function, the solution is inside the core, if such core results nonempty.

**Shapley value**
The Shapley value of a cooperative game is an allocation method which supplies a single cost allocation $\varphi$ able to satisfy certain axioms. The solution proposed by Shapley, in fact, was defined in such a way that it was possible to verify some properties, here illustrated.

**Symmetry**
An allocation $\varphi$ is defined symmetric if, taken two players $i$ e $j$, such as:

$$c(S \cup i) = c(S \cup j) \quad \forall \ S \subseteq N \setminus \{i,j\}$$  \hspace{1cm} (16),

then:

$$\varphi_i = \varphi_j$$  \hspace{1cm} (17).

**Dummy player**
A player $i$ is a *dummy player* if:

$$c(S \cup i) = c(S) + c(i) \quad \forall \ S \subseteq N \setminus \{i\}$$  \hspace{1cm} (18),

In case of a dummy player, Shapley value of such player is equal to his stand alone cost, i.e.:

$$\varphi_i = c(i)$$  \hspace{1cm} (19).

**Additivity**
An allocation $\varphi$ is defined additive if, given two games $u$ and $v$ and given the sum game $(u+v)$ defined from:

$$(u + v)(S) = u(S) + v(S) \quad \forall \ S \subseteq N$$  \hspace{1cm} (20),

it states that:

$$\varphi_i(u + v) = \varphi_i(u) + \varphi_i(v) \quad \forall i \in N$$  \hspace{1cm} (21).
Additivity condition is useful when we want to divide the realization of a project in different phases, or, vice versa, we want to unify different fulfilments, maintaining unvaried the assignment of costs to the users.

Theorem (Shapley, 1953): for every set $N$ there is only one method of cost allocation that is symmetric, additive and do not produce any advantage and disadvantage to a dummy player. Such method takes the name of Shapley value and it is equal to

$$
\varphi_i = \sum_{S \subseteq N-1} \frac{|S|! \left(|N - S| - 1\right)}{|N|!} [c(S + i) - c(S)]
$$

(22)

where

- $|S|$ cardinality of coalition $S$, i.e. number of players belonging to coalition $S$; for example $S = \{a,b\} \rightarrow |S| = 2$
- $|N|$ cardinality of the Grand Coalition, which is equal to the number of players of the game, i.e. $n$.

Shapley value is based on marginal contribution that every player can add to possible coalition (Fragnelli, 2010). According to Young (1985) it can be interpreted as the average of marginal contribution that every player would add to the Grand Coalition if it was formed by one player per time.

If the game is subadditive, Shapley value guarantees the total covering cost i.e. it satisfies the equation (10), but it is not sure that it is included inside the core. It belongs to the core if the characteristic function is concave, as expressed by (11).

**The nucleolus**

If it is fundamental the existence of a point solution inside the core, if it exists, then Shapley value does not satisfy such request. The most important unique method defined in order to result inside the core, in case in which this is a nonempty, is the so called nucleolus. Such nucleolus, defined by Schmeidler (1969), is based on the idea to select an allocation that makes the least-well-off coalition as well-off as possible (Young, 1985). In fact, the research of a point inside the core starts from the assumption to select an allocation which make the maximum discontent of a coalition as the minimum possible.

The problem is to agree on a meaning of "well-off". As reported by Young (1985) we can say that coalition $S$ is better off than $T$, relative to an allocation $x$, if

$$
c(S) - \sum_{i \in S} x_i > c(T) - \sum_{i \in T} x_i
$$

(23).

We define excess of $S$ relative to $x$ the quantity:
\[ e(x, S) = c(S) - \sum_{i \in S} x_i \]  

(24),

and the nucleolus represents the cost allocation that minimise the maximum excess \((Young, 1994)\), valuable using the linear programming according to the following imputation:

\[
\begin{align*}
\max \varepsilon \\
\text{s. t. } e(x, S) \geq \varepsilon \quad \forall S \neq N \\
\sum_{i \in N} x_i = c(N)
\end{align*}
\]  

(25).

In the restrictions the Grand Coalition is excluded because its excess is always null.

If there is a point solution \(x\) to the problem (25) then this is the nucleolus, otherwise it is necessary to use some algorithms, for example Kopelowitz's algorithm \((1967)\), which allows to supply a solution.

The idea of the nucleolus is to find a solution in the core that is "central" in the sense of being as far away from the boundaries as possible \((Young, 1985)\). In case of a two players game the core is represented by a segment and its medium point corresponds to the nucleolus. Moreover, even if the core is empty, the method of nucleolus supplies a solution.

Nucleolus per-capita
A reasonable variant of the nucleolus is to define the excess of a coalition on a per capita basis:

\[ e(S) = \frac{c(S) - \sum_{i \in S} x_i}{|S|} \]  

(26).

Under such condition, the solution of the related problem of linear programming expressed by (25) supplies a \textit{per capita nucleolus} or \textit{normalized nucleolus} \((Grotte, 1970)\).

4.2.2. Conclusions

To conclude such theoretical treatise, it is necessary to say that the only disadvantage of CGT is to require much information in presence of many players. In fact, it is opportune to value for all possible coalitions the relative associated
cost and the number of coalition of the game increase exponentially in function of the number of players: for \( n \) players the coalitions are equal to \( 2^n - 1 \).

The necessity to estimate minimal costs for every coalition is a prerogative of CGT. In fact, although we need a method of cost sharing for the more efficient alternative to the system, it is necessary to estimate also the costs associated to every coalition of the game, forasmuch they represent the parameters necessary to a fair, just and efficient allocation among the players.

### 4.2.2.1. Application to complex systems

Hereafter some applicative studies of cost allocation for complex systems, based on CGT methods, are presented. The examples proposed are listed according to the complexity of the problem and of the analysed system. Firstly it is illustrated the case of Tennessee Valley Authority, considered one of the first applicative cases of Game theory; then it is considered the application realized to the airport of Birmingham. Finally it is analysed the cost sharing realized for two complex water systems: the Swedish region of Skane and a municipal water system.


**Tennessee Valley Authority (TVA, 1938)**

One of the first CGT application to real systems is represented from the case of Tennessee Valley Authority (TVA), from 1938, i.e. previous to the publication of the book of Von Neumann e Morgenstern (1944).

TVA was instituted in the thirties to stimulate the economic activity in the mid-southern USA. One of its main objective was to build a series of dams and reservoirs along the Tennessee River in order to generate hydroelectric power, control flooding, and improve navigational and recreational uses of the waterway. Economists, analysing costs and benefits of the project observed that there was no completely obvious way to attribute costs to these purposes, because the system is designed to satisfy all of them simultaneously.

The concepts that they devised to deal with this problem foreshadow modern ideas in game theory, and the formulas of allocations used are still in use (in a different way) from different agencies of control of water systems among which “Bureau of Reclamation in the United States Department of the Interior”.

The costs shown in Table 11 have been evaluated by TVA for the construction of a reservoir for three different operators: navigation \([n]\), flood control \([f]\), power \([p]\). It can be noted as such information reflect the structure of the characteristic function of a cooperative game.
Cost allocation used by the American authority was not based on mathematical formulas but on so-called “evaluation” (TVA, 1938). On the impossibility to adopt any formal allocation among the known one at present, it was used an allocation criterion that is precursor of ACA method, whose results were rounded off through “evaluations” (Ransmeier, 1942).

**Airport landing fees (Littlechild & Thompson, 1977)**

Landing taxes of airplanes on airports are often established to cover costs of construction and maintenance of runways. The cost of construction of a runway is determined from the size of the biggest plane which lands on a specific airport.

In general, planes which will use the runway can be grouped in $t$ disjoint subsets \{N_1, N_2, …, N_t\} according to the length of runway necessary to land, so that the airplanes of the subset N_i require a runway equal to $c_i$ with $c_i < c_{i+1}$ (Figure 10).

![Figure 10. Scheme of a landing runway](image)

For this kind of problems it is showed that Shapley value of every plane corresponds to sharing cost obtained in the following way (Fragnelli, 2010): the cost of the first part of runway is divide among all the planes, because everyone uses it; the cost of the second part, $[c_2-c_1]$, is divided among the planes of subsets \{N_2, N_3, …, N_t\}, and so on, until the cost of the last part, $[c_{t-1}-c_t]$, which is divided among the planes of subset N_t that are the only in using it.
So, the Shapley value formula becomes:
\[
\phi_i = \phi_{i-1} + \frac{c_i - c_{i-1}}{|N_i \cup N_{i+1} \cup \ldots \cup N_1|}
\]

A real case of this kind of game has been analysed for the airport of Birmingham for which data of landing of different aeroplanes have been calculated for the years 1967-1968. From these data, chargeable costs to every class of aircraft and the relative allocation of costs applying the method of Shapley have been calculated (*Littlechild & Thompson, 1977*) (Figure 11).

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Number of Aircraft Landings</th>
<th>Annual Capital Cost</th>
<th>Charges (Shapley Value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fokker Friendship 27</td>
<td>42</td>
<td>65,909</td>
<td>4.88</td>
</tr>
<tr>
<td>Viscount 900</td>
<td>9,555</td>
<td>76,725</td>
<td>5.06</td>
</tr>
<tr>
<td>Hawker Siddeley Trident</td>
<td>203</td>
<td>65,200</td>
<td>10.30</td>
</tr>
<tr>
<td>Britannia</td>
<td>303</td>
<td>97,200</td>
<td>10.86</td>
</tr>
<tr>
<td>Caravelle VI R</td>
<td>151</td>
<td>97,439</td>
<td>10.92</td>
</tr>
<tr>
<td>BAC 111 (500)</td>
<td>1,315</td>
<td>90,142</td>
<td>11.13</td>
</tr>
<tr>
<td>Vanguard 950</td>
<td>505</td>
<td>102,496</td>
<td>13.40</td>
</tr>
<tr>
<td>Comet 4B</td>
<td>1,128</td>
<td>104,849</td>
<td>15.07</td>
</tr>
<tr>
<td>Britannia 300</td>
<td>151</td>
<td>113,322</td>
<td>44.36</td>
</tr>
<tr>
<td>Corvair Corronado</td>
<td>112</td>
<td>115,440</td>
<td>60.61</td>
</tr>
<tr>
<td>Boeing 707</td>
<td>22</td>
<td>117,679</td>
<td>162.24</td>
</tr>
</tbody>
</table>

**Figure 11. Data of the airport of Birmingham**

**Municipal Cost Sharing (Young et al., 1982)**

During the eighties Young et al. (1982) realized an interesting study using different CGT cost allocation methods considering the water system of region Skane in the Southern Sweden.

Such region is formed by 18 provinces whose water supply is guaranteed from three sources of supply: a groundwater source and two independent pumping systems which distribute the resource coming from the two lakes Vombsjon and Ringsjon (Figure 12)

During the '70s, the local authorities realized that water supply in use was not sufficient to guarantee the resource for future demands. So, different measures were analysed to increase water system, as the realization of new pipes, the increase of the pumping capacity and of the use of the groundwater resource.

On the basis of such scenario, Young et al. (1982) analysed the relative cooperative game with the aim to apply the different methods to allocate the modernization cost of system among the different provinces of the Swedish region.
The first problem was to identify the players. Considering every province as an independent player, it should be calculated the value of the function of cost for every possible coalition, i.e. $2^{18} - 1 = 262'143$. But it was clearly impossible. So, the provinces were grouped in 6 players, where the areas of similar characteristic were unified, in reference with the presence of a common water system, with the geography and the hydrology. (Figure 12).

For every coalition the least costs necessary to satisfy the demand including the future requests has been evaluated. The costs linked to water transport and water treatment have been calculated in function of the flow and of the distance from the source, through empirical formulations present in appendices of the article of Young et al. (1982).

The results obtained are present in the following Figure 13, where the letters that represent the single players are separated with commas just in case the coalition
do not present economies of scale, so, the value of the function cost has been evaluated from the sum of costs of the single players.

<table>
<thead>
<tr>
<th>Group</th>
<th>Total Cost</th>
<th>Group</th>
<th>Total Cost</th>
<th>Group</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>21.95</td>
<td>AHK</td>
<td>40.74</td>
<td>AHIKL</td>
<td>48.95</td>
</tr>
<tr>
<td>H</td>
<td>17.08</td>
<td>AHL</td>
<td>43.22</td>
<td>AHKM</td>
<td>60.27</td>
</tr>
<tr>
<td>K</td>
<td>10.91</td>
<td>AH, M</td>
<td>55.56</td>
<td>AH, T</td>
<td>63.72</td>
</tr>
<tr>
<td>L</td>
<td>15.88</td>
<td>AH, T</td>
<td>56.67</td>
<td>AHL, M</td>
<td>64.03</td>
</tr>
<tr>
<td>M</td>
<td>20.81</td>
<td>A, K, L</td>
<td>48.74</td>
<td>AHL, T</td>
<td>65.23</td>
</tr>
<tr>
<td>T</td>
<td>21.88</td>
<td>A, KM</td>
<td>41.40</td>
<td>AHI, MT</td>
<td>74.13</td>
</tr>
<tr>
<td>AH</td>
<td>34.69</td>
<td>A, K, T</td>
<td>54.84</td>
<td>A, K, LM</td>
<td>63.95</td>
</tr>
<tr>
<td>A, K</td>
<td>32.86</td>
<td>A, L, T</td>
<td>59.81</td>
<td>A, K, MT</td>
<td>72.27</td>
</tr>
<tr>
<td>A, L</td>
<td>37.83</td>
<td>A, MT</td>
<td>61.34</td>
<td>A, MT</td>
<td>71.41</td>
</tr>
<tr>
<td>A, M</td>
<td>42.76</td>
<td>HKL</td>
<td>27.26</td>
<td>HKL, M</td>
<td>48.01</td>
</tr>
<tr>
<td>A, T</td>
<td>43.93</td>
<td>HKM</td>
<td>42.55</td>
<td>HKL, T</td>
<td>49.24</td>
</tr>
<tr>
<td>HK</td>
<td>22.96</td>
<td>HK, T</td>
<td>44.94</td>
<td>HKM, T</td>
<td>59.35</td>
</tr>
<tr>
<td>HL</td>
<td>23.69</td>
<td>HL, M</td>
<td>45.81</td>
<td>HLMT</td>
<td>61.41</td>
</tr>
<tr>
<td>H, M</td>
<td>37.89</td>
<td>HL, T</td>
<td>46.98</td>
<td>HLMT</td>
<td>56.61</td>
</tr>
<tr>
<td>H, T</td>
<td>39.06</td>
<td>H, MT</td>
<td>56.49</td>
<td>AHKL, T</td>
<td>79.91</td>
</tr>
<tr>
<td>K, L</td>
<td>25.79</td>
<td>K, LM</td>
<td>42.01</td>
<td>AHKL</td>
<td>69.75</td>
</tr>
<tr>
<td>K, T</td>
<td>22.89</td>
<td>K, MT</td>
<td>48.77</td>
<td>AHKLMT</td>
<td>77.32</td>
</tr>
<tr>
<td>LM</td>
<td>31.10</td>
<td>LMT</td>
<td>51.46</td>
<td>AKLMT</td>
<td>73.97</td>
</tr>
<tr>
<td>L, T</td>
<td>37.86</td>
<td>HKMT</td>
<td>66.49</td>
<td>AHKLMT</td>
<td>81.82</td>
</tr>
<tr>
<td>MT</td>
<td>39.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values in Skr $\times 10^4$.

Figure 13. Characteristic function

Once obtained the characteristic function of the game, different allocations of costs of the Grand Coalition have been evaluated (Table 16).

<table>
<thead>
<tr>
<th>Cost Allocation [10^6 Skr (Swedish Kroner)]</th>
<th>A</th>
<th>H</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportional to the population</td>
<td>10.13</td>
<td>21.00</td>
<td>3.19</td>
<td>8.22</td>
<td>34.22</td>
<td>7.07</td>
</tr>
<tr>
<td>Proportional to demand</td>
<td>13.07</td>
<td>16.01</td>
<td>7.30</td>
<td>6.87</td>
<td>28.48</td>
<td>12.08</td>
</tr>
<tr>
<td>ACA</td>
<td>19.54</td>
<td>13.28</td>
<td>5.62</td>
<td>10.90</td>
<td>16.66</td>
<td>17.82</td>
</tr>
<tr>
<td>Shapley value</td>
<td>20.01</td>
<td>10.71</td>
<td>6.61</td>
<td>10.37</td>
<td>16.94</td>
<td>19.18</td>
</tr>
<tr>
<td>Nucleolus</td>
<td>20.35</td>
<td>12.06</td>
<td>5.00</td>
<td>8.61</td>
<td>18.32</td>
<td>19.49</td>
</tr>
<tr>
<td>Nucleolus per-capita</td>
<td>20.03</td>
<td>12.52</td>
<td>3.94</td>
<td>9.07</td>
<td>18.54</td>
<td>19.71</td>
</tr>
</tbody>
</table>

Table 16. Cost allocation

Efficient and equitable impact fees for urban water system (Lippai & Heaney, 2000)

The research of Lippai & Heaney (2000) refers to the determination of water urban rates through the use of principles of CGT. Through the results the authors put in evidence that in water rates the CGT costs allocation methods are more efficient with respect to the traditional methods based on a volumetric sharing.
The study considers a hypothetical water urban system fed by only one source that supplies three users: two residential blocks of low (user L) and middle density (user M) plus a warehouse (user W). It has assigned the water demand to every user and has defined the geometric scheme of the water system (Figure 14).

![Figure 14. Water scheme of reference](image)

Every possible coalition of user is characterized from a different position in the system and different water demands; this will have an effect on development and on dimensions of the water system and also on the relative cost function. To define the characteristic function of game it was considered for every coalition the least cost by the simulation program WinPipes belonging to software EPANET, which allows to minimize pressure drop, and by an optimization model to define the optimal economic solution.

The construction cost considers:
- the cost of the distribution system piping;
- the cost of the storage tank;
- the cost of the domestic and fire pumping system.

The characteristic function is present on Figure 15.

![Figure 15. Characteristic function](image)

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>Construction Cost Summary (in Dollars) of Optimal c(S) Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item description</td>
<td>MLW</td>
</tr>
<tr>
<td>Distribution system piping</td>
<td>481,844</td>
</tr>
<tr>
<td>Storage tank</td>
<td>345,690</td>
</tr>
<tr>
<td>Domestic and fire pumping systems</td>
<td>345,000</td>
</tr>
<tr>
<td>Total</td>
<td>1,002,254</td>
</tr>
</tbody>
</table>
Cost allocation has been valued using the ACA method which produces the results present on Table 17.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Cost [M$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>227</td>
</tr>
<tr>
<td>M</td>
<td>188</td>
</tr>
<tr>
<td>W</td>
<td>668</td>
</tr>
</tbody>
</table>

Table 17. Cost allocation for 3 players

It was also valued cost allocation considering the system constituted by only two players, unifying the residential blocks in only one player. In such case the cost allocation is the following.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Cost [M$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM</td>
<td>376</td>
</tr>
<tr>
<td>W</td>
<td>707</td>
</tr>
</tbody>
</table>

Table 18. Cost allocation for 2 players

If we compare the two allocations it is evident how the second one associates a higher cost for user W. So, we doubt whether one of the two solutions is the appropriate one. The answer of the authors is the same we have to give every time it is necessary to choose an allocation method of costs among the many one, i.e. the choice depends on the context of the game.

In our case if we consider the realization all over again of a water system, on which users have the same priorities of use of resource, then it is right to consider the players in an autonomous way; so the first allocation will be the appropriate one. But, if we consider the realization of a project in which two users have previously chosen some agreement of cooperation and a third one decides to enter into the same later, then the second solution will be preferable.
5. The optimization model WARGI

To study and analyze a water resources system in an integral form it is necessary to use mathematical and processing models which shall represent interrelationships among different elements: rivers, reservoirs, groundwater, water pipes, water users, etc. As expressed by Rizzoli & Young (2007) software systems that integrate models, or databases or other decision aids, and package them in a way that decision makers can use are commonly referred to as Decision Support Systems (hereafter DSS).

The practice of developing and using mathematical models of physical systems became common in most of the physical sciences with the advent of computers. Computer modelling is common used both as a method for scientists to test hypotheses and so better understand the way such systems function, and also as a predictive tool for those who manage such systems. Modelling provides a rapid means of investigating the expected response of a system to possible future changes, by undertaking the necessary computations which are commonly complex and data intensive (Rizzoli & Young, 1997).

As said by Heinz et al. (2007) the principal advantages of computer models for water systems are:

- they force us to be specific in representing our understanding of a system and identify gaps in our knowledge;
- they allow us to assess if simplified representations of uncertain aspects are likely to be adequate; and
- they allow us to apply our current knowledge to evaluate management alternatives.
Approaches to modelling river basins are typically simulations and optimization. Hydrologists, engineers, economists and other social scientists are to be involved in developing these models, with increasing involvement from stakeholders to ensure that models address their concerns and can be understood and trusted by diverse interests in a basin (Lund & Palmer, 1997; Palmer et al., 1999; Sechi & Sulis, 2009).

Simulation models are used to examine and evaluate specific “what if” scenarios, consisting of particular management decisions under particular scenario conditions (such as water demands or climate). Simulation models are relatively precise surgical tools for examining very specific conditions. They are excellent for exploring precise and specific management policies, and for exploring the ability of our quantitative understanding to mimic field behaviour.

While simulation models can estimate the effects of specific alternative water management strategies, hydro-economic optimization tools can identify promising combinations of diverse actions within natural and human-made constraints, such as availability of water resources and statutory rules. Optimization models help to identify “what’s best” in a broad sense, for refinement and testing with detailed simulation studies and negotiations.

As the number of options increases, simulation modelling alone cannot examine anything remotely close to all possible alternatives. Searching over large complex solution spaces for promising combinations of solutions requires optimization modelling. Optimization models typically employ a simpler formulation of the system than simulation models. Nevertheless, optimization models have their own limitations, requiring simplifications to accommodate optimization solution algorithms. Combined simulation-optimization methods allow optimization models to identify promising combinations of options, with simulation modelling to test and refine optimization model results (Lund & Ferreira 1996, Sechi & Sulis, 2009).

5.1. WARGI

Within such work DSS WARGI (Water Resources System Optimization Aided by Graphical Interface) (Sechi & Zuddas, 2000; Manca et al., 2004; Sechi & Sulis, 2009), developed from CRIFOR (Centro di Ricerca e Formazione delle Reti - Networks Investigation and Formation Centre) of the University of Cagliari, has been used.

This is a software based on an optimization approach developed for multi reservoirs and multi users water resource systems which allow to construct in a graphic interactive mode a system of study and to insert required data (hydrological, hydraulic, infrastructural, economic, etc.). On the basis of introduced information, WARGI can create an apposite file, codified according to MPS standard (Mathematical Programming Standard), which can be read and solved by many solvers on the market. For this research an optimization approach
in linear programming (LP) has been used utilizing solver CPLEX (http://www.ilog.com/products/cplex/).

In this research, WARGI is used to achieve the best water resource system performances. Using it we are able to calculate the least cost related to every coalition of the cooperative game to be analyze, and consequently to value its characteristic function.

![Initial interface of WARGI](image)

**Figure 16. Initial interface of WARGI**

Use of program WARGI is organized into following steps:

1. system representation through icons;
2. data entry (hydrological, hydraulic, infrastructural, economic...) in every element of water system (nodes and arcs);
3. saving water system in a file IDR and MPS file generation;
4. problem resolution with creation of file OUT by solver;
5. graphic display of results.

Minimum time step which is possible to use, corresponds to one month, a conventional interval used from the majority of water resources system models. However it is possible to personalize such process by choosing a bigger one, provided that it corresponds to a submultiple of 12 months.

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5.1.1. System Representation

Representation of water resources system is shown through a graph made of nodes and arcs, and every one of them represents an element of physical system of study. Such elements can be arranged and interconnected on a suitable area of program through the usage of particular icons (Figure 17).

![Figure 17. Area and icons for graph creation](image)

Elements that are possible to represent in WARGI are the following:

- reservoirs;
- civil, industrial and irrigational demand;
- hydroelectric power stations;
- confluences (which represent diversion dams or interconnection nodes);
- groundwater;
- pump station, desalinization plants, wastewater treatment plants and treatment plants;
- hydraulic connexions (which represent water bodies or water pipes).
For water balance it is important to consider spills from reservoirs, from confluences and from groundwater which are not represented on the graph but they flow in a fictitious node called “sea node”.

From sea node different arcs lead off other fictitious arcs which reach every system user. These represent the so-called deficit arcs, necessary to satisfy continuity equation in every demand node. On such node there is a transit of energy flow in default respect on water demand that the water system cannot produce. They represent possible water deficit for every user.

In Figure 18 an example of a graph is visible and it represents an hypothetic water system.

![Figure 18. Example of graph in WARGI](image)

### 5.1.2. Data entry

Entry of information for every water system element is generated through popup windows (from Figure 19 to Figure 22). Depending on the element, different fields shall be drawn up which allow to shape different elements from an hydraulic-hydrological (users’ demands, hydrologic input, groundwater recharge, evaporations from reservoirs, ...) infrastructural (dimensions of work, maximum and minimum capacity, pumping program,...) economical (data of costs, deficit and benefits,...) and environmental (minimum vital run off flow, infiltration, ...) point of view.
Figure 19. Reservoir window

Figure 20. Irrigational demand window
Every water system element can be considered in two different ways: operative or in project, depending if such element exists or it has to be inserted. Depending on the choice, different information shall be required.

For economic aspects, data required are:

- construction cost;
- operating cost;
- energetic cost for pumping.

Moreover it shall be indicated:

- spilling cost: cost linked to an extra resource which is removed from water system and transferred to “sea node” (only for node reservoir and node confluence);
- interperiod transfer benefit: negative cost linked to the resource which is kept on time in reservoirs (only for nodes reservoirs);
- deficit cost: cost linked to the amount of resource not given to demand (only for user nodes);
- benefit linked to possible hydroelectric power stations (only for node hydroelectric power station).

### 5.1.3. MPS file creation

The optimization model is based on a reproduction of a conceptual scheme of water system of study through the use of a MPS file which codifies the related linear programming problem.

Mathematical problem consists on minimizing the value of the objective function (OF) subject to a series of restrictions depending on data of water system elements. Objective function is subject of:

- equation of continuity to nodes;
- satisfaction of minimum and maximum capacity for nodes and arcs;
- positive variables.

Moreover, the version of program WARGI used for such survey considers other limitations which shall be analyzed afterwards.

Universal format MPS, developed in 1960 from IBM, is a largely diffused format and compatible from the majority of mathematical programming software and it was created to solve LP problems.

### 5.1.4. Optimization process

OF optimization process is assigned to solver CPLEX; this is a commercial programming engine for mathematical optimization problems. It was originally developed by R.E. Bixby and sold via CPLEX Optimization Inc., which was acquired by ILOG in 1997 that was finally acquired by IBM in January 2009. The software solves integer programming problems, very large linear programming problems, quadratic programming problems, and has recently added support for problems with convex quadratic constraints. It includes a pre-solve algorithm for
problem size reduction, sophisticated branching and cutting-plane strategies and feasibility heuristics.

Such program can read LP problem contained in MPS file, then it optimizes the OF and inserts optimization results in another file of extension “OUT”.

5.1.5. Display of results

The last step of the use of program WARGI consists in visualization of results.

WARGI can read the OUT file, which contains a solution to LP problems created by the solver. It can also represent results through simple graphs giving water flows related to every water system element for every time step (Figure 23 and Figure 24). Moreover, such model can create an appropriate file in text format representative of water flows which can be utilized by the user for his necessities.

![Figure 23. Stored volumes in a reservoir](image)

5.1.6. WARGI changes for CGT methodology

During researching activity some changes have been made to WARGI program to adapt better to CGT approach exigencies. Windows related to users nodes have been modified to insert other restrictions to OF which consider a minimum amount of water demand to supply.
Such procedure agrees with what established by some European Hydrologic Management Plan. For example in Spain the decree “Instrucciones de Planificación Hidrológicas” (Water Planning Instructions) (MMARM, 2008), that has the purpose to establish criteria to elaborate hydrologic plans for Spanish river basins, defines the so-called “niveles de garantía” (guarantee levels) for every different kind of water user. These represent the entity of maximum admitted deficit for every demand, expressed as a percentage of monthly/yearly water requests. They are different depending on water user typology:

For urban and industrial users:
- the deficit in a month cannot be more than 10% of the corresponding monthly request;
- in ten consecutive years, the sum of deficit cannot be more than 8% of the yearly request;

for irrigational user:
- the deficit in a year cannot be more than 50% of the yearly request;
- in two consecutive years, the sum of deficit cannot be more than 75% of the yearly request;
- in ten consecutive years, the sum of deficit cannot be more than 100% of the yearly request.

Figure 24. Water flow in entry for an irrigational demand
In order to consider a minimum amount of resource for every user, the restrictions inserted in WARGI are such as:

- for deficit related to monthly request:

\[ x_{ij} \leq \alpha_i R_{ij} \tag{28}. \]

where:

- \( x_{ij} \): water flow on deficit arc for i-user node in j-period;
- \( R_{ij} \): water demand of i-user node in j-period;
- \( \alpha_i \): entity of maximum admitted deficit for i-user node.

- for deficit related to yearly request:

\[ \sum_{j=1}^{n} x_{ij} \leq \alpha_i \bar{D}_i \tag{29} \]

where

- \( \bar{D}_i \): average of yearly water demand for i-user node;
- \( n \): time step related to maximum admitted deficit (1 year: \( n=12 \); 2 years: \( n=24 \), 10 years: \( n=120 \)).

\( \bar{D}_i \) is evaluated as:

\[ \bar{D}_i = \frac{\sum_{i=1}^{n} R_i}{n} \tag{30} \]

Italian legislation does not provide any limitation relative to the allowed maximum deficit. But on the present study it was hypothesized that deficit would
be null, so $\alpha=0$. Consequently the OF shall be subjected to other restrictions, represented by total supply of water users.

*For more details about WARGI see Salis et al. (2006).*
6. Water system examined

In this chapter we analyze the Sardinia water system, studying the regional multipurpose system and its different downstream sectors. So we describe in details the Flumendosa–Campidano system, one of the most important multipurpose schemes of the island, chosen for the application of the cost allocation methodology.

The principal source of information and data here reported is the Management Plan for River Basin District of Sardinia (RAS, 2010), version March 2010.

6.1. Territorial background

Sardinia is placed in the centre of the western basin of the Mediterranean Sea (Figure 25) and it is extended in a surface of 24,000 km² with a population of 1,648,000 residents. The climate is prevalently Mediterranean, characterized of a long period of dryness in summer and from mild and rainy winters with isolated frosts.

The Sardinia hydrography is typical of Mediterranean regions. The main bodies of water which present perennial flow are reported in Table 1; the most important Sardinian river is the Tirso, followed by Flumendosa.

The other bodies of water are characterized by torrential regime, mainly due to the narrow closeness between the mountains and the coast. Such bodies have prevalently big slopes in the majority of their course and are subject to flood in autumn and to periods of low water during the summer with the possibility of consecutive months of drought.

Hydrographical system presents important anthropic changes with many infrastructures that modify the natural waterways; in fact all the lakes of the
island, except lake Baratz, are artificial and from them the water supply system for water users is originated.

Figure 25. Mediterranean area

<table>
<thead>
<tr>
<th>Denomination</th>
<th>Length [km]</th>
<th>Basin [km²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tirso</td>
<td>153,60</td>
<td>3365,78</td>
</tr>
<tr>
<td>Flumendosa</td>
<td>147,82</td>
<td>1841,77</td>
</tr>
<tr>
<td>Fluminimannu</td>
<td>95,77</td>
<td>1779,46</td>
</tr>
<tr>
<td>Cedrino</td>
<td>77,18</td>
<td>1075,90</td>
</tr>
<tr>
<td>Taloro</td>
<td>67,71</td>
<td>495,02</td>
</tr>
<tr>
<td>Coghinas</td>
<td>64,40</td>
<td>2551,61</td>
</tr>
<tr>
<td>Liscia</td>
<td>51,83</td>
<td>570,74</td>
</tr>
<tr>
<td>Temo</td>
<td>47,71</td>
<td>839,51</td>
</tr>
</tbody>
</table>

Table 19. Principal Sardinia rivers.
Source CEDOC (2010)
6.2. Sardinian water system

Water supply system of Sardinia uses, almost totally, surface resources stored and regulated by artificial reservoirs. These have the task to protect from floods and in some cases to produce hydroelectric energy. Groundwater is used only for limited local needs.

The regional territory is divided in seven hydrographical zones (Figure 27). It is also considered another system, the number 8, constituted by two reservoirs utilized only to flood protection: the Santa Vittoria dam on Mogoro River and the Monte Crispu dam on Temo River (Table 20).

As reported in Chapter 2, Regional Law n°19 of 6.12.2006 introduce the concept of “multipurpose water system”, named SIMR; this system supplies “wholesale”
water for every downstream sector as civil, irrigational, industrial and hydroelectric demands.

<table>
<thead>
<tr>
<th>System</th>
<th>Denomination</th>
<th>Basin [km²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sulcis</td>
<td>1646</td>
</tr>
<tr>
<td>2</td>
<td>Tirso</td>
<td>5372</td>
</tr>
<tr>
<td>3</td>
<td>Nord Occidentale</td>
<td>5402</td>
</tr>
<tr>
<td>4</td>
<td>Liscia</td>
<td>2253</td>
</tr>
<tr>
<td>5</td>
<td>Posada – Cedrino</td>
<td>2423</td>
</tr>
<tr>
<td>6</td>
<td>Sud Orientale</td>
<td>1035</td>
</tr>
<tr>
<td>7</td>
<td>Flumendosa – Campidano – Cixerri</td>
<td>5960</td>
</tr>
<tr>
<td>8</td>
<td>Dam for flood control</td>
<td></td>
</tr>
</tbody>
</table>

Table 20. Sardinian water systems
Multipurpose section is managed by ENAS; civil section is organized in only one regional ATO (see Paragraph 2.2) on which there is only one operator: Abbanoa S.p.A; the irrigational sector is organized in nine land-reclamation syndicates and other agricultural areas run by ENAS. ASI and ZIR syndicates organize the supply water service for industrial demands, while ENEL S.p.A. and in a small way ENAS have the licence to manage hydroelectric power plants.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENAS</td>
<td>Multipurpose, irrigational and hydroelectric</td>
</tr>
<tr>
<td>ABBANOA S.p.A.</td>
<td>Civil</td>
</tr>
<tr>
<td>Land-reclamation syndicates</td>
<td>Irrigational</td>
</tr>
<tr>
<td>ASI and ZIR industrial Syndicates</td>
<td>Industrial</td>
</tr>
<tr>
<td>ENEL</td>
<td>Hydroelectric</td>
</tr>
</tbody>
</table>

Table 21. Water service operators

The general structure of the regional water system is illustrated in Figure 28.
Figure 28. Sardinia water system structure - Source Management Plan
6.2.1. Regional multipurpose water system

The SIMR is a very complex system which can be summarized as follows:

- 32 dams for a total of:
  - total volume: 1.992 Mm$^3$;
  - regulation volume: 1.870 Mm$^3$;
  - authorized regulation volume: 1.497 Mm$^3$;
- 25 diversion dams;
- more than 1000 km of water transfer works, as:
  - almost 800 km of pipelines;
  - almost 50 km of water tunnels;
  - almost 200 km of canals;
- 47 pumping stations in a total of 70 MW of installed power;
- 5 hydroelectric power plants for a total of 47.5 MW of power.

Infrastructures that belong to SIMR have been included in different “hydraulic schemes” (Table 22) grouping to the same scheme all the water works that aim to satisfy water needs of the same territory.

<table>
<thead>
<tr>
<th>Hydraulic systems</th>
<th>Hydraulic schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - Sulcis</td>
<td>1-A Mannu di Narcao</td>
</tr>
<tr>
<td></td>
<td>1-B Rio Palmas – Flumentepido</td>
</tr>
<tr>
<td>2 - Tirso</td>
<td>2-A Taloro</td>
</tr>
<tr>
<td></td>
<td>2-B Torrei</td>
</tr>
<tr>
<td></td>
<td>2-C Tirso – Mogoro – Fluminimannu di Pabillonis</td>
</tr>
<tr>
<td>3 - Nord – Occidentale</td>
<td>3-A Mannu di Pattada – Alto Tirso</td>
</tr>
<tr>
<td></td>
<td>3-B Coghinas – Mannu di Porto Torres</td>
</tr>
<tr>
<td></td>
<td>3-C Alto e Medio Temo – Cuga – Bidighinzu – Mannu di Ozieri</td>
</tr>
<tr>
<td></td>
<td>3-D Mannu di Sindia</td>
</tr>
<tr>
<td>4 - Liscia</td>
<td>4-A Liscia – Podrongiano</td>
</tr>
<tr>
<td></td>
<td>4-B Pagghiolu</td>
</tr>
<tr>
<td>5 - Posada – Cedrino</td>
<td>5-A Posada</td>
</tr>
<tr>
<td></td>
<td>5-B Cedrino</td>
</tr>
<tr>
<td>6 - Sud – Orientale</td>
<td>6-A Alto Flumendosa – Sa Teula</td>
</tr>
</tbody>
</table>
With regards to the amount of wholesale resource delivered to downstream users, we report in the Table 23 the volumes supplied from SIMR for the year 2009.

<table>
<thead>
<tr>
<th></th>
<th>Volume [Mm$^3$]</th>
<th>Percentage [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil demand</td>
<td>230,03</td>
<td>38</td>
</tr>
<tr>
<td>Irrigational demand</td>
<td>341,9</td>
<td>57</td>
</tr>
<tr>
<td>Industrial demand</td>
<td>27,32</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>599,25</td>
<td></td>
</tr>
</tbody>
</table>

Table 23. Delivered volumes in 2009

6.2.2. Civil sector

According to what established in Regional Law 29/1997 that fulfilled the Galli Law 36/1994, it was defined a unique ATO coinciding with the regional territory whose management, starting from 2006, was given to Abbanoa S.p.A. enterprise.

Regional ATO is organized in 8 districts, as shown in Figure 29; inside them there is the drinking distribution system that supplies residential and tourist sites of the island.

Civil demand mainly uses the resource supplied from SIMR, and in some case it uses the water stored in little reservoirs managed directly from Abbanoa S.p.A.

6.2.3. Irrigational sector

Irrigational demand represents the biggest user of water resource on the island. In Sardinia, irrigation is managed from 9 land-reclamation syndicates (Figure 30), that control an irrigable surface equivalent to 185,916 hectares and irrigated surface of 53,108 hectares (reference to year 2007).
Figure 29. Civil districts

The syndicates, as well as guarantee water supply to associated users, aim to achieve the following objectives:

- management and maintenance of irrigational water distribution systems;
- valorisation and rational use of water resources;
- defence and safeguard of the soil;
- defence of the environment and valorisation of territory.

Irrigational water system is constituted essentially by transport and distribution pipelines and it includes pumping stations, where necessary.

The main source of supply for every syndicate is represented by the wholesale resource acquired from ENAS and, in some case, depending on geographical localization, by the resource taken from own sources like groundwater sites, rivers and water springs.
Besides the irrigational districts of land-reclamation syndicate, we have to add some agricultural areas directly managed by ENAS.

6.2.4. Industrial sector

The industrial sector manages both the water supply system for industrial plants grouped in ASI and ZIR, and the service of waste water collection and treatment. The Industrial demand includes 13 syndicates and 9 hand-crafts areas (Figure 31).
Industrial demand partially uses the resource supplied from SIMR because, apart from its local sources of resource as wells and water springs, it uses treatment plants for water reuse and in some cases small desalination plants.

Figure 31. Industrial and hand-craft areas.
Source: www.sardegnasuap.it

6.2.5. Hydroelectric sector

Sardinian hydroelectric system includes 19 hydroelectric plants (Table 24).
Table 24. Hydroelectric plants in Sardinia

Most of such plants are managed by ENEL S.p.A, with only one of them that is reversible. Other plants belong to SIMR, in particular to Flumendosa-Campidano-Cixerri multipurpose scheme, and so these are directly managed by ENAS

6.3. The Flumendosa - Campidano multipurpose water system

The cost allocation methodology has been applied considering a portion of the regional water system. We chose the so-called Flumendosa-Campidano system, one of the principal multipurpose water systems of Sardinia.
The system belongs to the river basin number 7, the Flumendosa – Campidano-Cixerri, and it is constituted by three multipurpose schemes, 7A-B-E (see Table 22) that supply the three macro users of the centre-southern zone of the island, i.e.:

- civil demand of district 1 and 3 of regional ATO, with the metropolitan area of Cagliari constituted by almost 400,000 residents;
- irrigational demands of the “Sardegna Meridionale” Land-Reclamation Syndicate (hereafter CdB-SM) the biggest of the nine Sardinian Syndicates, constituted by 32 districts with a total irrigable surface of 60,000 hectares.
- irrigational districts managed by ENAS;
- industrial demands “CASIC” of Sarroch and Macchiareddu.

The principal resources of the system are represented by flows from Flumendosa, Mulargia and Fluminimannu River, to which are added resources coming from smaller water bodies. These resources are regulated by several infrastructures (7 reservoirs and 7 diversion dams) and delivered to users through a net of pipelines, tunnels, canals and pumping stations.

Moreover, the partial exploitation of the resource coming from the system number 2 Tirso it is also possible, through the use of an interconnection with a pumping station in case of water scarcity. The basin of Tirso uses the resource stored in lake Omodeo, the biggest of Sardinia and one of the biggest in Europe.

Also multipurpose schemes 6A and 1A have been inserted in the analysis, since even if they do not belong to the Flumendosa – Campidano system, they are hydraulically interconnected to such system and so they influence its behaviour: the system 6A is situated upstream of the scheme 7A, near the spring of Flumendosa river, while the scheme 1A draws resource on scheme 7E.
In the table below, we report the water demands of the system. Final users have been grouped in different centres of demand according to the scheme reported on PSURI.

<table>
<thead>
<tr>
<th>User</th>
<th>Scheme</th>
<th>Centre of demand</th>
<th>Name</th>
<th>Request [Mm³]</th>
<th>Operator</th>
<th>Total user [Mm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>6A</td>
<td>D72</td>
<td>Ogliasta</td>
<td>1,3</td>
<td>Abbanoa</td>
<td>101,5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D72-Flut</td>
<td>Touristic</td>
<td>1,5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ogliasta</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7A</td>
<td>D41</td>
<td>Sarcidano1</td>
<td>0,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D45</td>
<td>Sarcidano2</td>
<td>7,8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D57</td>
<td>Gerrei</td>
<td>0,5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D44</td>
<td>Santu Miali</td>
<td>2,8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D48</td>
<td>Donori</td>
<td>18,4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D51-a</td>
<td>San Michele</td>
<td>31,2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>User</td>
<td>Scheme</td>
<td>Centre of demand</td>
<td>Name</td>
<td>Request [Mm³]</td>
<td>Operator</td>
<td>Total user [Mm³]</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>D51-b</td>
<td>23,2</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>D51-Flut</td>
<td>3,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7E</td>
<td></td>
<td></td>
<td>D58</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D58-Flut</td>
<td>1,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1A</td>
<td></td>
<td></td>
<td>D54</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irrigational</td>
<td>6A</td>
<td>D75-76</td>
<td>Ogliastro</td>
<td>11,27</td>
<td>Ogliastrea Land-reclamation syndicate</td>
<td>93,26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>7A</td>
<td></td>
<td></td>
<td>D39</td>
<td>0,99</td>
<td>ENAS</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D42</td>
<td>9,15</td>
<td>Sardegna Meridionale Land-reclamation syndicate</td>
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</tr>
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<td></td>
<td></td>
<td>D43</td>
<td>21,9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7B</td>
<td></td>
<td></td>
<td>D46</td>
<td>9,02</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D47</td>
<td>13,32</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D50-D60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D52</td>
<td>6,92</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D59</td>
<td>4,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7E</td>
<td></td>
<td></td>
<td>D49</td>
<td>4,78</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D53</td>
<td>0,3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial</td>
<td>6A</td>
<td>D73</td>
<td>Arbatax</td>
<td>1</td>
<td>Industrialization pole Tortoli - Arbatax</td>
<td>16,00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7E</td>
<td></td>
<td></td>
<td>D55</td>
<td>10</td>
<td>CASIC</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>D56</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>210,76</td>
</tr>
</tbody>
</table>

Table 25. Water users of Flumendosa – Campidano system. 
Source: PSURI and SMGSIR.

Afterwards, we describe the different multipurpose schemes which constitute water system chosen for our survey.
6A - Alto Flumendosa – Sa Teula

Hydraulic scheme 6A is located in the central-eastern part of the island and it is formed principally from five dams linked in series, four managed by ENEL and the downstream one by ENAS.

The main dam is called “Bau Muggeris” which delimits a reservoir of 63 Mm³ near to the spring of Flumendosa river. It also collects run-offs of Bau Mela and Bau Mandara rivers, that are delimited from the homonymous dams which realize two little reservoirs of 0,24 Mm³ and 0,31 Mm³. The three dams are connected in series through tunnels that deliver the resource to two hydroelectric plants realized in series. The second plant is linked to a little reservoir on Sa Teula river, of 0,14 Mm³ of volume, connected to a third hydroelectric plant which discharges the resource directly to the river. This is delimited downstream from another dam, called “Santa Lucia” which forms a reservoir of 3,6 Mm³ of volume. The water system supplies the irrigational demand of Ogliastra land-reclamation syndicate (D75-76), the residential and tourism civil demands (D72, D72-Flut) and the industrial demands of the area of Tortoli – Arbatax (D73).

7A - Medio e Basso Flumendosa – Fluminimannu

The scheme consists of hydraulic infrastructures which permit to use the resources of Flumendosa, Mulargia, Flumineddu and Fluminimannu rivers.

The main resource is represented by the water stored in the “Flumendosa” reservoir of 263 Mm³, delimited from the dam “Nuraghe Arrubiu”, and in the “Mulargia” reservoir of 320 Mm³, delimited from the “Monte Su Rei” dam; they are connected in series through a tunnel. The system receives also the resource from the reservoir of 1,44 Mm³ on the river Flumineddu, a Flumendosa affluent. Moreover the resources from the low course of Flumendosa river, derived from
“S’Isca Rena” diversion dam, are delivered to the Mulargia reservoir through the pumping station named “Basso Flumendosa”.

In the north of the lake Flumendosa, there is placed the “Ponte Maxia” diversion dam which allows, through “Villanova Tulo” pumping station, the derivation of resource to civil and irrigational demands of the surrounding zone (D39, D41, D45). They can also be supplied by the reservoir on Fluminimannu river of 12,24 Mm³, delimited from “Is Barrocus” dam.

From “Mulargia” reservoir a pipeline, in service of surrounding civil demand (D57), and a tunnel, which terminates in the “Sarais” node, are originated. “Sarais” node supplies northern districts of CdB-SM (D42) and then the resource can continue in western direction to the “Adduttore” canal or in southern direction to the “Mulargia-Cagliari” aqueduct.

“Adduttore” canal supplies the northern irrigational districts of CdB-SM (D43) and civil demands of surrounding centres (D44). It ends into the little reservoir “Sa Forada”, delimited from the homonymous dam which guarantees a volume of 1,33 Mm³. Here also the run-offs coming from Tirso river basin, through a water interconnection which includes the “Sardara” pumping station, can be delivered. “Sa Forada” reservoir is connected through a tunnel to the “Casa Fiume” diversion dam on the river Fluminimannu. The two dams constitute the central node of the water scheme Flumendosa-Campidano.

The “Mulargia-Cagliari” aqueduct feeds the potable water treatment plant of Donori, in service of surrounding centres (D48). In the final line it forks and delivers the resource to the two potable water treatment plant for the residential and tourist civil demands of metropolitan area of Cagliari (D51a, D51b, D51-Flut).
They can also be supplied with the resource stored in the little reservoir on the river Corongiu of 4.3 Mm³, directly managed by Abbanoa.

**Figure 35. Water scheme 7A (2)**

**7B - Campidano: Fluminimannu-Mannu di Monastir**

The scheme takes origins from the “Sa Forada” and “Casa Fiume” reservoirs and consists of all the works that deliver the resources coming from the scheme 7A and from the Fluminimannu and Mannu rivers.

From “Casa Fiume” diversion dam two irrigational canals, called “Ripartitore Sud-Est” and “Ripartitore Est-Ovest”, are originated.

The first runs along Campidano area in direction North West - South West and feeds the central irrigational districts of CdB-5M (D47, D50-60). Near the town of Monastir the canal takes on the resource intercepted from the “Monastir” diversion dam on Mannu river and more downstream it is interconnected with the scheme 7E through a bidirectional pipeline. It goes on as a pipeline and it joins in the “San Lorenzo” node with a branch of “Mulargia-Cagliari” aqueduct. It follows in eastern direction and can feed the civil demand (D51b) and irrigational demand.
(D52) of the surrounding area through two pumping stations, named “Simbirizzi-civil” and “Simbirizzi-irrigation”; so it finishes in the “Simbirizzi” reservoir of 30,3 Mm³. The reservoir, in service of the irrigational user, apart from storing the resource delivered from the upstream system, is used to take on the treated waste water of the urban area of Cagliari, given through the "Is Arenas" pumping station.

The second canal from “Casa Fiume” diversion dam is developed in the west zone, it feeds the surrounding irrigational districts (D59) and finally it forks in the “Sud-Ovest” and “Nord-Ovest” canals. The first follows in direction south and it is connected to the “Cixerri” reservoir, while the second is developed until the limits of the Oristano province feeding northwest districts of the CdB-SM (D46).

From “Sa Forada” reservoir the “Nuovo Ripartitore Sud-Est” pipeline is originated. The work represents the following of the transference line of resources from Tirso to Campidano. Moreover, such work results linked to the “Mulargia-Cagliari” aqueduct through an interconnection with possibilities of bidirectional functioning, which increases the flexibility of water transfers in the system. Along its course this pipeline feeds the central districts of CdB-SM (D47, D50-60) and in “San Lorenzo” node it links with the “Ripartitore Sud-Est” and with a branch of “Mulargia-Cagliari” aqueduct.

![Figure 36. Water scheme 7B](image-url)
7E - Basso Cixerri – Fluminimannu - Santa Lucia

The scheme 7E uses the resources of the river Cixerri, integrated with those from diversion dams on Fanaris, Santa Lucia and Monti Nieddu rivers.

The diversion on Fanaris river is connected to the second branch of “Sud-Ovest” canal that feeds the western districts of CdB-SM (D49) and ends in “Cixerri” reservoir.

The “Cixerri” reservoir of 24 Mm³, delimited from the “Genna Is Abis” dam, is the principal node of the scheme. It is functional to the users’ needs of the south-west
area of Flumendosa-Campidano system and it permits the delivery of the resource to the system 1A.

“Cixerri” reservoir is linked, through the homonymous pumping station, to the “Cixerri-Macchiareddu” pipeline. This can also receive the resource derived from the diversion dam on Santa Lucia river. The “Cixerri-Macchiareddu” pipeline feeds the surrounding irrigational districts (D53) and it enters in the “Macchiareddu” node; from this node a bidirectional interconnection to the scheme 7B, included the “Macchiareddu” pumping station, is originated.

From “Macchiareddu” node also the industrial aqueduct takes origin. It is managed by the industrial syndicate “CASIC” and it is developed in direction south where it feeds the industrial pole of Macchiareddu and Sarroch (D55, D56) and the residential and tourist civil demand of the surrounding area (D58, D58-Flut).

The scheme is completed with the diversion dam on Monti Nieddu river which, through the homonymous pumping station, is interconnected to the final branch of the industrial aqueduct and allows to increase the available resource for users.

1A - Mannu di Narcao

Such a scheme is functional to supply civil demands (D54) directly linked to “Bau Pressiu” reservoir of 8,5 Mm³ on Mannu of the Narcao river.

During water emergency in the period 1985 – 1990 the interconnection with the scheme 7E was realized to permit the water transfer from the “Cixerri” reservoir through the “Sulcis” pumping.
7. Methodology of cost allocation

7.1. Premise and objectives

As we saw in a previous chapter, CGT allows us to value cost sharing among the participants of a project in a fair, efficient and impartial way, respecting the principles of individual acceptability and common agreement.

So, the objective of the research was to develop a methodology of cost allocation applied to a water resources system using the CGT methods. For this reason we can give an original contribution both to mathematical sciences and to hydro-economic modelling.

This kind of methodology, based on mathematical science, allows us to give an adequate justification of adopted criteria, in order to determine a cost division which can be shared by all users. Moreover, through the cooperation among participants it is possible to maximise the efficiency of water resource management, a very important goal in Mediterranean water systems which are characterized by water scarcity.

7.2. Description of methodology

The methodology for the application of CGT to water resources systems consists essentially of the following steps:

- **Water system description**: functional definition of the water system and evaluation of its different aspects (hydrologic, hydraulic, infrastructural, economic, etc.).
Cooperative game planning: identification of the players and coalitions, analysis of priorities, etc.

Characteristic function (c.f.) valuation: set up of the optimization model, calculation of the minimum associated cost for every coalition and valuation of the c.f. of the game;

Game solution: application of CGT methods necessary to share the costs among the players.

In the first step we have to identify all the important aspects for the description and characterization of water system, in particular the hydrologic, hydraulic and infrastructural aspects. During this phase we have to add the economic analysis necessary for the calculation of c.f. of the cooperative game: it will be indispensable to individuate the typology of cost which characterizes the water system and which we want to share among the different water users.

In the case of construction cost allocation of new infrastructures, it is essential that the analysis of function costs are relative to the realization of works. This type of approach has been analyzed in the application described in Deidda (2009) with regards to the adaptation of the water resources system in the Turia river basin, in Spain, carried out in the international project “Azioni Integrate Italia-Spagna”. In this context the dimensions necessary to guarantee some standard service levels, defined by the Spanish Normative in the “Instrucciones de Planificacion Hidrologica” (see Chapter 5), have been evaluated and the costs among the users have been divided using a unique allocation criterion, the Shapley value.

Nevertheless, quite often the situation is different: the system is already equipped and the exigency is to allocate the management costs. In this case it is necessary to consider the charges relative to the routine and supplementary maintenance costs, to the adaptation and substitution of the works and to the energy costs for the pumping. The application which will be illustrated hereinafter concerns the allocation of this typology of costs.

In the second phase of the methodology we have to identify the players and to set up the cooperative game. The players can represent both individual users and sets of them, as in case of users belonging to an unique macrodemand (for example irrigational or industrial syndicate, urban municipalities, etc...). This step is important because we have to remember that the number of players affects the complexity of the problem as given $n$ players the number of coalitions that we have to analyze is equal to $2^n - 1$.

Then there is the most important phase of the methodology: the definition of c.f. of the game. According to the definition, c.f. is constituted from the set of minimum costs associated to all the possible coalitions of players. We have to insist on a concept: the necessity to value the minimum costs for every coalition is a prerogative of the CGT. In fact even if we want to share the costs of the Grand Coalition, which represents the best situation for the system in condition of subadditivity, it is also essential to value costs associated to subcoalitions, since such costs represent the necessary parameters that permit a fair and efficient sharing among the players. In our case the minimum cost of a coalition is defined
as the sum of the management costs of the “minimum” set of infrastructures necessary to totally satisfy the water request of the users in that coalition. This is possible thanks to the use of the optimization model WARGI which, in function of the costs linked to every infrastructure, optimizes the management of the system minimizing the OF representative of every coalition. Depending on the entity and on the spatial and temporal variability of water demand of the users in a coalition, it will vary the number and the kind of infrastructures to be used and the possible amount of resources to be pumped; consequently it will vary the sum of management costs of the infrastructures and so the associated costs to the specific coalition.

The last phase, i.e. the game solution, needs the application of the CGT methods to share the costs of the water system among the players. We preferred to use a subset solution defining the core of the game, in such a way we can individuate for every player the minimum and maximum attributable cost. Using this kind of solution we have the chance to provide to the decision-maker an admissible range of costs related to every user, which results a useful tool to define water rates.

Developing the methodology outlined above, two different approaches have been considered; these have different peculiarities in the application of CGT to water systems.

At the beginning, it was defined the minimum set of infrastructures in service of every coalition associating to the relative users the maximum priority of use of water system. In such a circumstance the users in the coalitions have priority in using the resources and infrastructures of the system to satisfy their needs. This approach has determined some problems when high water requests were considered. In fact, as we will see later, the condition of subadditivity of the c.f. was not satisfied when the water demand of the system increased and so the Grand Coalition was not the most efficient solution. In this circumstance it was not possible to define the core of the game.

In a second moment a totally different approach was used, because it was valued the allocation costs assigning to the users in coalition the minimum priority. In this case it is necessary to determine for every coalition, not the direct cost of use of infrastructures but the marginal one: This is the management cost linked to the use of available resource and of residual capacity of transportation and regulation infrastructures in relation to the preliminary usage of the water system by the players not belonging to the coalition being examined. In this case the determination of reference system configuration in which we have to realize the optimization (and consequentially the c.f evaluation.) is very hard-working. In fact for every coalition it is necessary to operate before the optimization of the system considering the only users out of the coalition in order to examine the entities of the resource and the capacity of the infrastructures available for the users in coalition. Operating with marginal costs it is shown that the c.f. of the game is subadditive, so the most efficient solution for the system is represented by the Grand Coalition and as a consequence we have the advantage of always being able to define the core of the cooperative game.
To better illustrate the cost allocation methodology we present hereinafter the results obtained on a exemplification of the Flumendosa-Campidano water system; instead, in the following chapter, we will apply the methodology to the whole complex system.

7.3. **Application to a simplified water system**

**Water system description**

The methodology application and the comparison of results for the two different approaches have been done for the water system in Figure 39, which essentially represents a simplification of the Flumendosa - Campidano multi purpose water system.

![Figure 39. Water system](image-url)

The system is functional for the water supply of the three multi purpose macro demands: civil, irrigational and industrial. These can be supplied using the resource coming from two reservoirs in series and from a diversion dam situated downstream, these works are representative of the set of infrastructures of the multi purpose regional schemes 7A and 7B. Moreover, it is possible the partial usage of the run offs of the near basin through a pumping station which corresponds to Tirso-Flumendosa water interconnection. The sum of the pipelines, canals and tunnels of the real system is reproduced by the set of arcs in service of the three users. The two reservoirs have a capacity of 260 Mm³ the first and 320
Mm$^3$ the second. To simplify the calculation we did not apply upper bounds for the other elements of the graph.

The three users have different water demands, whose percentage share during the hydrologic year (from October to September) is reported in Table 26 in accordance with what reported in PSURI (RAS, 2006a). The civil and industrial users are characterized by a constant monthly demand, while the irrigational user is characterized by a bigger demand in the dry period (April-September), with a peak in the month of July (Figure 40).

<table>
<thead>
<tr>
<th></th>
<th>Irrigational</th>
<th>Civil and Industrial</th>
</tr>
</thead>
<tbody>
<tr>
<td>October</td>
<td>4,05</td>
<td>8,33</td>
</tr>
<tr>
<td>November</td>
<td>2,11</td>
<td>8,33</td>
</tr>
<tr>
<td>December</td>
<td>1,46</td>
<td>8,33</td>
</tr>
<tr>
<td>January</td>
<td>1,62</td>
<td>8,33</td>
</tr>
<tr>
<td>February</td>
<td>1,62</td>
<td>8,33</td>
</tr>
<tr>
<td>March</td>
<td>2,92</td>
<td>8,33</td>
</tr>
<tr>
<td>April</td>
<td>6,16</td>
<td>8,33</td>
</tr>
<tr>
<td>May</td>
<td>10,86</td>
<td>8,33</td>
</tr>
<tr>
<td>June</td>
<td>18,64</td>
<td>8,33</td>
</tr>
<tr>
<td>July</td>
<td>24,8</td>
<td>8,33</td>
</tr>
<tr>
<td>August</td>
<td>18,31</td>
<td>8,33</td>
</tr>
<tr>
<td>September</td>
<td>7,45</td>
<td>8,33</td>
</tr>
</tbody>
</table>

Table 26. Percentage division of water demand. Source PSURI

![Water demand division graph](image-url)
Hydrological inputs have been extracted from PSURI. The regional document provides the series of run offs for 53 years of reference on the basis of observed data in the main hydrographical station of Sardinia. Three hydrological inputs, linked to the two reservoirs and to the diversion dam, that are relative to Flumendosa-Campidano system have been chosen. Moreover it is hypothesized that in case of water scarcity high water drawing (100 Mmc/year) from the adjacent river basin is possible.

The series of run offs are reported in the following figures.

![Figure 41. Hydrological input Dam 1](image)

![Figure 42. Hydrological input Dam 2 and Diversion Dam](image)
According to the hydrological series, the time-length chosen for the hydro-economic analysis is equal to 53 years, with a monthly time-step.

In relation to the economic aspects, as we said in the premise, we want to analyze management costs of the system considering maintenance costs of infrastructures and possible energy costs linked to pumping.

Regarding the maintenance costs, we used values reported on PSURI, that are estimated on the basis of the model in Chapter 2. The yearly maintenance cost of the works in the graph is due to the sum of the routine and supplementary maintenance costs of the corresponding infrastructures of the Flumendosa – Campidano system (Table 27).

<table>
<thead>
<tr>
<th>Graph element</th>
<th>Flumendosa - Campidano work</th>
<th>Routine maintenance cost (source PSURI) [€/year]</th>
<th>Supplementary maintenance cost (source PSURI) [€/year]</th>
<th>Total maintenance cost [€/year]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dam 1</td>
<td>“Nuraghe Arrubiu” Dam</td>
<td>180.000</td>
<td>343.748</td>
<td>523.748</td>
</tr>
<tr>
<td>Dam 2</td>
<td>“Monte Su Rei” Dam</td>
<td>180.000</td>
<td>318.878</td>
<td>498.878</td>
</tr>
<tr>
<td>Tunnel 1</td>
<td>“Flumendosa – Mulargia” Tunnel</td>
<td>2.959</td>
<td>61.372</td>
<td>64.331</td>
</tr>
<tr>
<td>Tunnel 2</td>
<td>“Mulargia – Sarais” Tunnel</td>
<td>4.962</td>
<td>111.642</td>
<td>116.604</td>
</tr>
<tr>
<td>Canal</td>
<td>“Adduttore” canal</td>
<td>130.617</td>
<td>223.334</td>
<td>353.951</td>
</tr>
<tr>
<td>Pipeline 1</td>
<td>“Mulargia – Cagliari” Aqueduct</td>
<td>191.314</td>
<td>682.802</td>
<td>874.116</td>
</tr>
<tr>
<td>Diversion dam</td>
<td>“Casa Fiume” diversion dam</td>
<td>10.000</td>
<td>4.000</td>
<td>14.000</td>
</tr>
<tr>
<td>Pipeline 2</td>
<td>Pipelines and canals</td>
<td>809.858</td>
<td>1.116.668</td>
<td>1.926.526</td>
</tr>
<tr>
<td>Pumping + Interconnection</td>
<td>“Sardara” Pumping + “Tirso – Flumendosa” Interconnection</td>
<td>351.128</td>
<td>522.851</td>
<td>873.979</td>
</tr>
</tbody>
</table>

Table 27. Maintenance costs

The pumping station, in addition to the maintenance cost, is subject to energy costs, which are linked to the amount of resource to be pumped.

From the analysis made by PSURI for the year 2006 the average energy unit price for pumping is equal to 0,05 €/m³. This value derives from the calculation realized
on 37 pumping stations in the regional territory in function of the average yearly volumes pumped and of the annual energy consumption. On that occasion an average unit price of kWh equal to 0,124 € had been adopted.

Thanks to what was reported in the more actualized SMGSIR (RAS, 2008) it is said that the average unit price of the kilowatt hour suffered an increase which determined a value equal to 0,183 €/kWh. So, the correspondent average energy unit price is changed to 0,07 €/mc. To simplify the calculation, in this survey we decided to choose this value as the unique energy unit price related to pumping stations.

<table>
<thead>
<tr>
<th>Graph element</th>
<th>Energy cost [€/mc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pumping</td>
<td>0,07</td>
</tr>
</tbody>
</table>

Table 28. Energy cost

Cooperative game planning

Once the different aspects of water system have been defined, we can start applying the cooperative game planning procedure.

In our case every user represents a single independent player, so the game coalitions are formed by the single players, as [civil], [irrigational] or [industrial]; by their partial aggregation, as [civil + irrigational], [civil + industrial] or [irrigational + industrial]; and by the Grand Coalition that, as we know, is constituted by all players, i.e. [civil + irrigational + industrial]. The aim of the game is to share among users the cost of the most efficient alternative for the system which allows their complete water demand satisfaction.

To apply the methodology we considered two water demand scenarios, as shown in Table 29. The first one is representative of the current situation of the Flumendosa – Campidano water system, while the second hypothesizes a situation of development of the irrigational water demand, according to what was reported in PSURI (RAS, 2006). Every scenario has been analyzed using the two different approaches described in the premise: different priorities, maximum and minimum, will be associated to users in the coalition.

<table>
<thead>
<tr>
<th>Yearly water demand [Mm³/year]</th>
<th>Scenario A</th>
<th>Scenario B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>Irrigational</td>
<td>70</td>
<td>200</td>
</tr>
<tr>
<td>Industrial</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>160</td>
<td>290</td>
</tr>
</tbody>
</table>

Table 29. Water demand scenarios
Optimization model set up

The minimum cost of every coalition is estimated optimizing the water system management through the DSS WAR GI for each one. For every coalition an MPS file is associated and inside it there is a related OF to be optimized. Every OF is differentiated from the others exclusively by the different water demands of users in a coalition.

Cost values considered in the OF are:
- management cost of the infrastructures, i.e. maintenance and energy cost;
- spilling cost;
- interperiodal transfer cost for reservoirs.

The OF optimization is constrained to:
- equation of continuity to nodes;
- maximum capacity for reservoirs nodes;
- total satisfaction of users in coalition (i.e. absence of deficit).

The spilling cost is related to the overabundance of resource which is eliminated from the system and transferred to the “sea node”. It is equal to zero for nodes with hydrological input, while for the others it takes a high value at will. In this way in the optimization phase it will be more convenient to transfer to the sea node the possible overabundant resource directly from the nodes with hydrological input; so it is possible to avoid the introduction inside the system of a surplus of resource which would determine a bigger use of the infrastructures and so a higher total cost.

Interperiodal transfer cost is linked to the resource stored in time in a reservoir. This cost is considered equal to a small value at will in order to minimally penalize the regulation capability of the reservoirs and at the same time to avoid the storing of unnecessary resource and to guarantee the minimum use of the infrastructures.

Inserting these two typologies of cost it is possible to use the minimum amount of resource assuring, as we want, the use of the minimum set of infrastructures necessary to satisfy the water system demands.

Moreover, the optimization model WAR GI allows one to assign a cost to the amount of the resource not given to the demands, the so called deficit cost. In this survey such a cost is equal to zero because the OF of every coalition is subject to the total users’ satisfaction.

For every coalition the optimization outputs will provide water flows in water system; then in function of them we will evaluate the infrastructures in service of each coalition and so the related management cost.

7.3.1. Maximum priority

As we said, in this case the minimum cost of each coalition is calculated by assigning to its players the highest priority of use of water system resources and infrastructures.
7.3.1.1. Scenario A

Characteristic function valuation

A synthesis of optimization results is reported in Table 30, in which the infrastructures used by each coalition are reported.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Infrastructures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dam 1</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Dam 2</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Tunnel 1</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Tunnel 2</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Canal</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Diversion dam</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pipeline 1</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Pipeline 2</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pumping + Interconnection</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
</tr>
</tbody>
</table>

Table 30. Infrastructures used by coalitions

**Maximum Priority – Scenario A**

By analysing the results we can observe that neither the Grand Coalition nor the others use all the water system infrastructures. The Grand Coalition is the only one which uses both dams, while for the others the resource stored in dam 2 only is enough. The priority use of the second work is due to the fact that it presents a lower maintenance cost in respect to the other. Moreover, we can note that among all the coalition in which there is a civil demand, only the one formed by a single player uses pipeline 1. In other cases civil demand cooperates with the other players in the coalition using the canal and the pipeline 2, because, this way, a lower management cost is determined. Finally, we observe that the pumping station is never used: this means that the system does not have a water deficit since it is not necessary to supplement resource from the adjacent basin.

On the basis of optimization outputs total management costs associated to every coalition are evaluated. These will be equal to the sum of maintenance costs of used infrastructures plus the possible pumping costs, both relative to the whole time-length. When an infrastructure is not constantly used every year, its maintenance cost will be considered for every year in any case, because the yearly maintenance, necessary to preserve the functionality of the work, will always be required.
Total management costs of coalitions are reported in Table 31: because energy costs are nil, due to the unused pumping station, management costs are equal to maintenance costs.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Yearly maintenance Cost [M€/year]</th>
<th>Total maintenance Cost (53 years) [M€]</th>
<th>Energy cost (53 years) [M€]</th>
<th>Total management Cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>1,55</td>
<td>82,15</td>
<td>0.00</td>
<td>82,15</td>
</tr>
<tr>
<td>Irrigational</td>
<td>2,97</td>
<td>157,41</td>
<td>0.00</td>
<td>157,41</td>
</tr>
<tr>
<td>Industrial</td>
<td>2,91</td>
<td>154,23</td>
<td>0.00</td>
<td>154,23</td>
</tr>
<tr>
<td>Irrigational – Industrial</td>
<td>2,97</td>
<td>157,41</td>
<td>0.00</td>
<td>157,41</td>
</tr>
<tr>
<td>Irrigational – Civil</td>
<td>2,97</td>
<td>157,41</td>
<td>0.00</td>
<td>157,41</td>
</tr>
<tr>
<td>Civil – Industrial</td>
<td>2,97</td>
<td>157,41</td>
<td>0.00</td>
<td>157,41</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>3,50</td>
<td>185,50</td>
<td>0.00</td>
<td>185,50</td>
</tr>
</tbody>
</table>

Table 31. Management costs of coalitions  
Max Priority – Scenario A

To simplify the calculation we did not make any cost actualisation, but by applying the common economic formulas this aspect will always be possible to consider.

Total management costs associated to coalitions defines the c.f. of the game (Table 32).

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Characteristic function [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>82,15</td>
</tr>
<tr>
<td>Irrigational</td>
<td>157,41</td>
</tr>
<tr>
<td>Industrial</td>
<td>154,23</td>
</tr>
<tr>
<td>Irrigational – Industrial</td>
<td>157,41</td>
</tr>
<tr>
<td>Irrigational – Civil</td>
<td>157,41</td>
</tr>
<tr>
<td>Civil – Industrial</td>
<td>157,41</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>185,50</td>
</tr>
</tbody>
</table>

Table 32. Characteristic function  
Maximum Priority – Scenario A
Consequently, we are able to verify if the Grand Coalition is the most efficient alternative for the water system, that means to check if it represents the best economical solution. For that we compare the cost of the Grand Coalition with those associated to all possible combinations of coalitions in order to estimate all the possible alternatives for water system. (Table 33).

<table>
<thead>
<tr>
<th>Combinations of Coalitions</th>
<th>Cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grand Coalition</td>
<td>185,50</td>
</tr>
<tr>
<td>[Civil] + [Industrial – Irrigational]</td>
<td>239,56</td>
</tr>
<tr>
<td>[Industrial] + [Civil – Irrigational]</td>
<td>311,64</td>
</tr>
<tr>
<td>[Irrigational] + [Civil – Industrial]</td>
<td>314,82</td>
</tr>
<tr>
<td>[Irrigational] + [Civil] + [Industrial]</td>
<td>393,79</td>
</tr>
</tbody>
</table>

Table 33. Efficiency of the Grand Coalition
Maximum Priority – Scenario A

From Table 33 we observe that the cost of Grand Coalition is the least and this shows the convenience of cooperation among the players for the system. Therefore, the next step is represented by the application of the solution methods of CGT necessary to share the cost of Grand Coalition among the three players.

**Game solution**

As we said, we chose a subset solution, estimating the core of the game, through which it is possible to define the set of allocations that have to respect the limits of maximum and minimum cost attributable to every player. Such boundaries are evaluated by applying the marginality and rationality principles in the correct way:

- the minimum value is equal to the marginal cost necessary to the single player to enter in the Grand Coalition; it can be calculated by subtracting the cost of the coalition constituted by other players from the cost of Grand Coalition;
- the maximum value is equal to the cost associated to the single player, i.e. the stand alone cost.

For the three water user we obtain the following boundary values.
### Table 34. Minimum and maximum costs for the players.

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value [M€]</th>
<th>Maximum value [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>min(Civ) = c(N) - c(Irr-Ind) = 28,09</td>
<td>MAX(Civ) = c(Civ) = 82,15</td>
</tr>
<tr>
<td>Irrigational</td>
<td>min(Irr) = c(N) - c(Civ-Ind) = 28,09</td>
<td>MAX(Irr) = c(Irr) = 157,51</td>
</tr>
<tr>
<td>Industrial</td>
<td>min(Ind) = c(N) - c(Civ-Irr) = 28,09</td>
<td>MAX(Ind) = c(Ind) = 154,23</td>
</tr>
</tbody>
</table>

Consequently, the core of the game is constituted by the set of allocations which respect, at the same time, both above-reported ranges and total recovery of the cost of Grand Coalition. Analytically the allocations have to solve the following system:

\[
\begin{align*}
\text{Civ + Irr + Ind} & = 185,50 \\
28,09 & \leq \text{Civ} \leq 82,15 \\
28,09 & \leq \text{Irr} \leq 157,51 \\
28,09 & \leq \text{Ind} \leq 154,23
\end{align*}
\]

If we express the boundary values in percentage we obtain:

### Table 35. Minimum and maximum percentage values for the players.

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value [%]</th>
<th>Maximum value [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>15,1</td>
<td>44,3</td>
</tr>
<tr>
<td>Irrigational</td>
<td>15,1</td>
<td>84,9</td>
</tr>
<tr>
<td>Industrial</td>
<td>15,1</td>
<td>83,1</td>
</tr>
</tbody>
</table>

Graphic representation of the core according to the triangular diagram presented in Chapter 3 is reported in Figure 43.

Moreover, in economic analysis linked to water resources it is interesting to express the boundary values that define the core of the game using unit cost per cubic metre of water delivered (Table 36).
Values shown in previous tables are the final results of the application of the methodology of cost allocation to the water system.

7.3.1.2. Scenario B

Another scenario characterized by a higher water demand was analyzed. It was chosen to “stress” the water system and to consequently consider the use of the pumping station.

Characteristic function valuation

Infrastructures used by coalitions are reported in Table 37.
### Table 37. Infrastructures used by coalitions

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dam 1</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
<tr>
<td>Dam 2</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Tunnel 1</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Tunnel 2</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Canal</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Diversion dam</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pipeline 1</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Pipeline 2</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pumping + Interconnection</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

Analysing the optimization output we can see that in this new scenario the water system run offs are not sufficient to satisfy some coalitions, so an extra resource through the use of the pumping station is required. This necessity belongs to every coalition in respect of the previous scenario have increased their water demand, i.e. those in which there is the irrigational user. The others, i.e. [civil], [industrial] and [civil + industrial], are characterized by the same results obtained in the previous scenario since their requests are the same and so the relative OF has not suffered changes.

In function of the infrastructures used we evaluate the management costs linked to every coalition. In this case it is necessary to evaluate before for every coalition the pumped volumes to determine the relative energy cost (Table 38).

### Table 38. Pumped volumes and energy cost

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Pumped volumes [Mmc]</th>
<th>Energy cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>0,00</td>
<td>0,00</td>
</tr>
<tr>
<td>Irrigational</td>
<td>595,19</td>
<td>41,66</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,00</td>
<td>0,00</td>
</tr>
<tr>
<td>Irrigational – Industrial</td>
<td>919,98</td>
<td>64,40</td>
</tr>
<tr>
<td>Irrigational – Civil</td>
<td>3,803,13</td>
<td>266,22</td>
</tr>
<tr>
<td>Civil – Industrial</td>
<td>0,00</td>
<td>0,00</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>4,312,42</td>
<td>301,87</td>
</tr>
</tbody>
</table>
Summing the two typologies of costs we determine the total management cost associated to every coalition. (Table 39).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>1,55</td>
<td>82,15</td>
<td>0,00</td>
<td>82,15</td>
</tr>
<tr>
<td>Irrigational</td>
<td>4,37</td>
<td>231,61</td>
<td>41,66</td>
<td>273,27</td>
</tr>
<tr>
<td>Industrial</td>
<td>2,91</td>
<td>154,23</td>
<td>0,00</td>
<td>154,23</td>
</tr>
<tr>
<td>Irrigational - Industrial</td>
<td>4,37</td>
<td>231,61</td>
<td>64,40</td>
<td>389,02</td>
</tr>
<tr>
<td>Irrigational - Civil</td>
<td>4,37</td>
<td>231,61</td>
<td>266,22</td>
<td>497,83</td>
</tr>
<tr>
<td>Civil - Industrial</td>
<td>2,97</td>
<td>157,41</td>
<td>0,00</td>
<td>157,41</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>4,37</td>
<td>231,61</td>
<td>301,87</td>
<td>533,48</td>
</tr>
</tbody>
</table>

Table 39. Management costs of coalitions
Max priority – Scenario B

The c.f. of the game is reported in the following table.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Denomination</th>
<th>Characteristic function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>Civ</td>
<td>82,15</td>
</tr>
<tr>
<td>Irrigational</td>
<td>Irr</td>
<td>273,27</td>
</tr>
<tr>
<td>Industrial</td>
<td>Ind</td>
<td>154,23</td>
</tr>
<tr>
<td>Irrigational – Industrial</td>
<td>Irr-Ind</td>
<td>389,02</td>
</tr>
<tr>
<td>Irrigational – Civil</td>
<td>Irr-Civ</td>
<td>497,83</td>
</tr>
<tr>
<td>Civil – Industrial</td>
<td>Civ-Ind</td>
<td>157,41</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>N</td>
<td>533,48</td>
</tr>
</tbody>
</table>

Table 40. Characteristic function
Max Priority – Scenario B

Comparing costs associated to the possible combinations of coalitions we check if, also for this scenario, the Grand Coalition is the most efficient solution for the system (Table 41).
### Combinations of Coalitions

<table>
<thead>
<tr>
<th>Coalitions</th>
<th>Cost (53 years) [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Irrigational] + [Civil – Industrial]</td>
<td>430,68</td>
</tr>
<tr>
<td>[Civil] + [Industrial – Irrigational]</td>
<td>471,17</td>
</tr>
<tr>
<td>[Irrigational] + [Civil] + [Industrial]</td>
<td>509,65</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>533,48</td>
</tr>
<tr>
<td>[Industrial] + [Civil – Irrigational]</td>
<td>652,06</td>
</tr>
</tbody>
</table>

**Table 41. Efficiency of the Grand Coalition**  
**Max Priority – Scenario B**

We observe that in this case the cost of the Grand Coalition is not the least. So it implies the inconvenience of not defining the core of the game.

**Game solution**

For this scenario the most efficient alternative for water system management is represented by the combination of [Civil] and [Industrial+Irrigational] coalitions. In this case the game solution proposed by CGT would associate to the civil demand its stand alone cost and would define a sub core for the other players. (Table 42).

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum [M€]</td>
</tr>
<tr>
<td>Irrigational + Industrial</td>
<td>389,02</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Civil</td>
<td>82,15</td>
</tr>
</tbody>
</table>

**Table 42. Allocation cost.**  
**Max priority – Scenario B**

This kind of solution determines a combined use of the infrastructures for the two coalitions. Let’s consider for example the stored volumes in Dam 2, as shown in Figure 44.
Adding the values of the two coalitions we obtain the total stored volumes in the reservoir.

Analyzing the graph in Figure 45 we can see that the reservoir, with a capacity of 320 Mm$^3$, is used over its maximum limit, which represents an infeasible solution.
Now let’s consider the run-off used by the two coalitions. Focusing on the hydrologic input linked to the second reservoir, we obtain the following graph.

![Figure 46. Available resource and used Dam 2. Max priority - Scenario B](image)

In Figure 46 the blue area represents the entity of the run offs available in correspondence of the Dam 2, while the green one represents the relative resource used by the two coalitions. We note that the green area is bigger than the other, this determines the second inadmissible condition of used resource being higher than the available one.

This behaviour is intrinsic to the characteristics of the planned cooperative game. In fact associating the maximum priority to the users in coalition, the infrastructures with low cost will be firstly assigned by the optimizer to a generic coalition; but in this way, all the coalitions will be under the best conditions and so possible interactions with other players out of coalition will not be considered. This can determine an over-use of infrastructures and/or an over-exploitation of resource. Under these conditions the Grand Coalition will result the unique correct alternative because it is not subject to the behaviour of any player out of coalition.

To conclude we can say that associating the maximum priority to the users in coalition, it is possible to use the allocation methodology described only in cases in which the Grand Coalition is the most efficient condition for the system. On the contrary apart from the impossibility to define the core of the game we would obtain inadmissible solutions.
7.3.2. **Minimum priority**

On the basis of what we observed before, we considered another approach which took care of the iterations among external and internal players to a coalition. For this reason we chose to associate different priorities of use of the water system: minimum for the water users in a coalition and maximum for the others.

In this way the lowest costs that form the c.f. of the game are going to be determined not from the direct costs of use of the infrastructures but from its marginal costs. These are determined in function of the available resource and of the residual capacities of the transportation and regulation works consequent to the preliminary exploitation of the system by players with a bigger priority. In this way, the definition of the optimal management of the system for the players in a coalition is influenced by the behaviour of the players out of it.

For example, let’s consider the coalition constituted by the civil demand. Supposing that the players out of coalition, i.e. the irrigational and industrial demand, use 80% of the capacity of a reservoir, so the civil demand will dispose of the remaining residual capacity of 20%. In case that such amount is not sufficient to satisfy the player, he will have the need to use another infrastructure.

The same evaluations have to be done for the estimation of the residual resource: the resource used by irrigational and industrial demand will be valued for each hydrological input, and then it will be subtracted from each run-off, defining new ones for the civil demand.

Constraints related to resource and infrastructures capacities, called “marginality constraints” are evaluated preliminarily optimizing the system management for the player that, from time to time, will be out of coalition.

The Grand Coalition will not be characterized by any marginality constraints because, not existing players out of it, such a coalition won’t be influenced in the exploitation of the system by anybody.

So in this different approach of cost allocation, the coalition formed by partial unions of players will be damaged and, on the contrary, the total users cooperation will be reward.

7.3.2.1. **Scenario A**

**Marginality constraints definition**

In the examined water system the only infrastructures with a limit in capacity are the reservoirs nodes and the possible water drawn from the near basin was hypothesized as unlimited. So the marginality constraints to be estimated are referred only by the residual capacity of the two dams and to the available resource linked to the reservoirs and to the diversion dam.
As we said, such values come from simple optimization procedures, in particular it is possible the use of the results of optimizations realized in the previous approach because the coalitions that we analyzed before are formed by the players that for this approach from time to time are out of coalition.

Hereinafter we propose the detailed procedure for the evaluation of the marginality constraints for the coalition formed by the civil demand that is consequently influenced by irrigational and industrial users.

**Residual capacity of the reservoirs**

**Dam1**

As we can see in Table 30 the group of players out of coalition (irrigational and industrial users) do not use the first reservoir, so the whole capacity of the work will be available for the civil demand and it is not necessary any constraint to be linked to it.

**Dam2**

As far as the second reservoir is concerned, we report the residual capacity available to the civil demand that is defined by the volumes represented in Figure 47 by the yellow area that is the complementary to the maximum capacity of the work (320 Mm³) respect to the stored volumes for the users out of coalition.

![Figure 47. Stored volumes and available volumes in Dam 2](image)

So, in the optimization procedures the coalition formed by the civil demand won’t dispose, for the whole period of analysis, the maximum capacity of the reservoir
but only a limited and variable volume as shown in Figure 48. Such storable volume will be inserted in WARGI to define the relative MPS file.

**Figure 48. Residual capacity Dam 2 (a)**

*Residual hydrological input*

Below the resource relative to the hydrological inputs used by players out of coalition is shown.
On the basis of the graphics in Figure 49 and in Figure 50, we value the residual resource available for the civil demand, defining new hydrological series.
Figure 51. Residual input Dam 1 (a)

Figure 52. Residual input Dam 2 (a)
Also this new hydrological input will be considered for the optimization process. Marginality constraints have also been evaluated for the other coalitions and are shown below.

*Residual capacity of the reservoirs*

**Dam1**
As for civil demand, analyzing Table 5 we can see that every group of players, who from time to time are out of coalition, do not use the reservoir; so its maximum capacity is available for all the coalitions without any constraints.

**Dam2**
For the second reservoir we obtain the following limits of capacity.
The coalition which results the most disadvantaged is that one formed by industrial demand. This is due to the fact that its relative players out of coalition (civil and irrigational demand) need a big use of the work.
Residual hydrological input

Input Dam1

Figure 56. Residual Input Dam 1 (b)

Figure 57. Residual Input Dam 1 (c)
Input Dam 2

Residual input Dam 2

Figure 58. Residual input Dam 2 (b)

Residual input Dam 2

Figure 59. Residual input Dam 2 (c)
Input Diversion dam

Figure 60. Residual input Diversion dam (b)

Figure 61. Residual input Diversion dam (c)
From the analysis of the graphs we note the presence of periods of zero input, especially for industrial user; this situation will condition the use of water infrastructures.

**Characteristic function valuation**

On the basis of marginality constraints optimization procedures, necessary to define the minimum set of infrastructures in service of every coalition, have been realized (Table 43).

Grand Coalition presents the same results seen before because the relative OF was not modified as it is not subject to any marginality constraint. For others coalitions it is clear that, with respect to the previous approach, there is an increase in the number of used infrastructures: for example all the coalitions, except the one formed by the industrial demand, use both the reservoirs. The industrial user, on the contrary to the first approach, use only the first reservoir: its behaviour is due to the constraints linked to the second reservoir. In fact, as shown in figure 20, it has a low residual capacity of the work and that determines the preferential utilization of the upstream dam. Moreover, also in this case, the support of resource through the pumping station for any coalition is not necessary.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Infrastructures</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dam 1</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Dam 2</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Tunnel 1</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Tunnel 2</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Canal</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Diversion dam</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pipeline1</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Pipeline2</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pumping + Interconnection</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
</tr>
</tbody>
</table>

Table 43. Infrastructures used by coalitions

Minimum Priority – Scenario A

Once the infrastructures in service of every coalition are defined, we can value management costs related to each coalition (Table 44), the c.f. of the game (Table 45) and the efficiency of the Grand Coalition (Table 46).
<table>
<thead>
<tr>
<th>Coalition</th>
<th>Yearly maintenance cost [M€/year]</th>
<th>Total maintenance cost (53 years) [M€]</th>
<th>Energy cost (53 years) [M€]</th>
<th>Total management cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>2,07</td>
<td>109,71</td>
<td>0.00</td>
<td>109,71</td>
</tr>
<tr>
<td>Irrigational</td>
<td>3,50</td>
<td>185,50</td>
<td>0.00</td>
<td>185,50</td>
</tr>
<tr>
<td>Industrial</td>
<td>3,00</td>
<td>159,00</td>
<td>0.00</td>
<td>159,00</td>
</tr>
<tr>
<td>Irrigational-Industrial</td>
<td>3,50</td>
<td>185,50</td>
<td>0.00</td>
<td>185,50</td>
</tr>
<tr>
<td>Irrigational - Civil</td>
<td>3,50</td>
<td>185,50</td>
<td>0.00</td>
<td>185,50</td>
</tr>
<tr>
<td>Civil - Industrial</td>
<td>3,50</td>
<td>185,50</td>
<td>0.00</td>
<td>185,50</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>3,50</td>
<td>185,50</td>
<td>0.00</td>
<td>185,50</td>
</tr>
</tbody>
</table>

Table 44. Management cost of coalitions
Minimum Priority – Scenario A

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Characteristic function [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>109,71</td>
</tr>
<tr>
<td>Irrigational</td>
<td>185,50</td>
</tr>
<tr>
<td>Industrial</td>
<td>159,00</td>
</tr>
<tr>
<td>Irrigational - Industrial</td>
<td>185,50</td>
</tr>
<tr>
<td>Irrigational - Civil</td>
<td>185,50</td>
</tr>
<tr>
<td>Civil - Industrial</td>
<td>185,50</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>185,50</td>
</tr>
</tbody>
</table>

Table 45. Characteristic function
Minimum Priority – Scenario A

<table>
<thead>
<tr>
<th>Combinations of Coalitions</th>
<th>Cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grand Coalition</td>
<td>185,50</td>
</tr>
<tr>
<td>[Civil] + [Industrial - Irrigational]</td>
<td>295,21</td>
</tr>
<tr>
<td>[Industrial] + [Civil - Irrigational]</td>
<td>344,50</td>
</tr>
<tr>
<td>[Irrigational] + [Civil - Industrial]</td>
<td>371,00</td>
</tr>
<tr>
<td>[Irrigational] + [Civil] + [Industrial]</td>
<td>454,21</td>
</tr>
</tbody>
</table>

Table 46. Efficiency of the Grand Coalition
Minimum Priority – Scenario A
Even in this case, the Grand Coalition is confirmed as the most efficient alternative.

**Game solution**

On the basis of the c.f. of the game, the boundary values of cost attributable to the players are the following ones.

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value [M€]</th>
<th>Maximum value [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>( \min(Civ) = c(N) - c(Irr-Ind) = 0,00 )</td>
<td>( \max(Civ) = c(Civ) = 109,71 )</td>
</tr>
<tr>
<td>Irrigational</td>
<td>( \min(Irr) = c(N) - c(Civ-Ind) = 0,00 )</td>
<td>( \max(Irr) = c(Irr) = 185,50 )</td>
</tr>
<tr>
<td>Industrial</td>
<td>( \min(Ind) = c(N) - c(Civ-Irr) = 0,00 )</td>
<td>( \max(Ind) = c(Ind) = 159,00 )</td>
</tr>
</tbody>
</table>

Table 47. Values of minimum and maximum cost for the players

Minimum Priority – Scenario A

Then, the core is represented by the set of the solutions of the following system:

\[
\begin{align*}
\text{Civ} + \text{Irr} + \text{Ind} &= 185,50 \\
0,00 &\leq \text{Civ} \leq 109,71 \\
0,00 &\leq \text{Irr} \leq 185,50 \\
0,00 &\leq \text{Ind} \leq 159,00
\end{align*}
\]

(32).

Expressing the boundary values in percentage we obtain:

<table>
<thead>
<tr>
<th>Players</th>
<th>Minimum value [%]</th>
<th>Maximum value [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>0,0</td>
<td>59,1</td>
</tr>
<tr>
<td>Irrigational</td>
<td>0,0</td>
<td>100</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,0</td>
<td>85,7</td>
</tr>
</tbody>
</table>

Table 48. Minimum and maximum percentage values for the players

Minimum Priority – Scenario A

We can note that the minimum values for the three players are the same and equal to zero. This is due to the fact that the costs of coalitions formed by two players are equal to the one associated to the Grand Coalition, because they use the same infrastructures and so applying the marginality principle we obtain a null minimum value. Moreover, irrigational demand is characterized by a maximum
value equal to the cent per cent of the cost of Grand Coalition since they have the same management costs.

Boundary values expressed in unit cost per cubic metre of water delivered are reported below.

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value [€/mc]</th>
<th>Maximum value [€/mc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>0,000</td>
<td>0,026</td>
</tr>
<tr>
<td>Irrigational</td>
<td>0,000</td>
<td>0,050</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,000</td>
<td>0,300</td>
</tr>
</tbody>
</table>

Table 49. Minimum and maximum unit costs for the players
Minimum Priority – Scenario A

Comparing these results with those of the previous approach, we note an increase in the range of boundary values, due to the raise of costs associated to all coalitions (except the Grand Coalition). Also from a graphic point of view we can note the expansion of the admissible area, as shown in Figure 62. The area represented in vertical lines correspond to the core of this game, while the smaller area, in oblique lines, symbolizes the core obtained before.

Figure 62. Allocation of core
Minimum Priority – Scenario A
7.3.2.2. Scenario B

Marginality constraints definition

With respect to scenario A there is a change in water demand for the irrigational user. This fact influences the marginality constraints only for the coalitions that have such users out of them, i.e. [Civil], [Industrial] and [Civil – Industrial]. For these coalitions related constraints are presented below, for the others the same obtained in precedence are confirmed.

Residual capacity of the reservoirs

Dam1

In this case it is necessary to evaluate for the above-mentioned coalitions, also the residual capacity of the first reservoir, shown below.

![Figure 63. Residual capacity Dam 1](image-url)
In correspondence with some periods, we register a low residual capacity of reservoirs, that will influence their use for users in coalition.

**Residual capacity of the reservoirs**

**Input Dam1**

---

**Figure 64. Residual capacity Dam 2 (d)**

**Figure 65. Residual input Dam 1 (d)**
Input Dam 2

Figure 66. Residual input Dam 2 (d)

Input Diversion dam

Figure 67. Residual input Diversion dam (d)
We can see that the three above-mentioned coalitions can benefit from a low hydrological input; this is due to the grand exploitation of the system by players out of coalition. In particular, run-off for industrial demand are null in correspondence with the two reservoirs, such demand won’t be rely on them for its supply. In these circumstances a big support of resources coming from the near basin through the pumping will be necessary.

**Characteristic function valuation**

The minimum set of infrastructures in service of coalitions is present in Table 50.

In this case, all the coalitions use the pumping station for the impossibility of the only use of regulations infrastructures due to the scarce hydrological input and to the low capacity of reservoirs. With respect to these previous situations, the necessity of the pumping station and the consequentially use of pipeline 2 induce the civil demand to avoid the use of pipeline 1, in order to obtain lower management costs. Moreover, as mentioned earlier, the supply for industrial demand is only possible via the resources coming from the near basin and from the diversion dam.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Infrastructures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dam 1</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Dam 2</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Tunnel 1</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
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<td>Tunnel 2</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Canal</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Diversion dam</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pipeline1</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
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<tr>
<td>Pipeline2</td>
<td>YES</td>
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<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Pumping + Interconnection</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

**Table 50. Infrastructures used by coalitions**

Minimum Priority – Scenario B

Hereinafter we present the pumped volumes and the relative costs (Table 51), total management costs related to coalitions (Table 52), the c.f. of the game (Table 53) and the efficiency of the Grand Coalition (Table 54).
### Table 51. Pumped volumes and energy cost
Minimum priority – Scenario B

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Pumped volumes [Mmc]</th>
<th>Energy cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>3.392,44</td>
<td>237,47</td>
</tr>
<tr>
<td>Irrigational</td>
<td>4.345,66</td>
<td>304,20</td>
</tr>
<tr>
<td>Industrial</td>
<td>509.29</td>
<td>35,65</td>
</tr>
<tr>
<td>Irrigational - Industrial</td>
<td>4.597,76</td>
<td>321,84</td>
</tr>
<tr>
<td>Irrigational - Civil</td>
<td>4292,178</td>
<td>300,45</td>
</tr>
<tr>
<td>Civil - Industrial</td>
<td>3.717,23</td>
<td>260,21</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>4.312,42</td>
<td>301,87</td>
</tr>
</tbody>
</table>

### Table 52. Management cost of coalitions
Minimum Priority – Scenario B

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>4,37</td>
<td>231,61</td>
<td>237,47</td>
<td>469,08</td>
</tr>
<tr>
<td>Irrigational</td>
<td>4,37</td>
<td>231,61</td>
<td>304,20</td>
<td>535,81</td>
</tr>
<tr>
<td>Industrial</td>
<td>2,82</td>
<td>149,46</td>
<td>35,65</td>
<td>185,11</td>
</tr>
<tr>
<td>Irrigational - Industrial</td>
<td>4,37</td>
<td>231,61</td>
<td>321,84</td>
<td>553,45</td>
</tr>
<tr>
<td>Irrigational - Civil</td>
<td>4,37</td>
<td>231,61</td>
<td>300,45</td>
<td>532,06</td>
</tr>
<tr>
<td>Civil - Industrial</td>
<td>4,37</td>
<td>231,61</td>
<td>260,21</td>
<td>491,82</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>4,37</td>
<td>231,61</td>
<td>301,87</td>
<td>533,48</td>
</tr>
</tbody>
</table>

### Table 53. Characteristic function
Minimum Priority – Scenario B

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Characteristic function [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>469,08</td>
</tr>
<tr>
<td>Irrigational</td>
<td>535,81</td>
</tr>
<tr>
<td>Industrial</td>
<td>185,11</td>
</tr>
<tr>
<td>Irrigational - Industrial</td>
<td>553,45</td>
</tr>
<tr>
<td>Irrigational - Civil</td>
<td>532,06</td>
</tr>
<tr>
<td>Civil - Industrial</td>
<td>491,82</td>
</tr>
<tr>
<td>Grand Coalition</td>
<td>533,48</td>
</tr>
</tbody>
</table>
Assigning the minimum priority to users in coalition also for the second scenario of water requests we can determine that the Grand Coalition is the most efficient solution and so, contrary to the previous approach, it is possible to apply the CGT solution methods.

**Game solution**

On the basis of c.f. of the game and applying suitably the rationality and marginality principles, the boundary values of attributable cost to players are the following.

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value [M€]</th>
<th>Maximum value [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>min(Civ) = c(N) – c(Irr-Ind) = -19,97</td>
<td>MAX(Civ) = c(Civ) = 469,08</td>
</tr>
<tr>
<td>Irrigational</td>
<td>min(Irr) = c(N) – c(Civ-Irr) = 41,66</td>
<td>MAX(Irr) = c(Irr) = 535,81</td>
</tr>
<tr>
<td>Industrial</td>
<td>Min(Ind) = c(N) – c(Civ-Irr) = 1,42</td>
<td>MAX(Ind) = c(Ind) = 185,11</td>
</tr>
</tbody>
</table>

The presence of the negative minimum value for civil demand is due to the fact that the costs associated to coalition [Irrigational - Industrial] is bigger than that one of Grand Coalition, so the marginality value for civil demand is lower than zero. In our research we cannot consider the negative values and so we hypothesize that the lowest value for a coalition is equal to zero. Moreover, the maximum cost attributable to the irrigational user is bigger than that one of Grand Coalition; so, also in this case, we modified the boundary value and we put it equal to the cost of the Grand Coalition.

The values which define the core are modified as follows:
The analytical expression of the core is the following:

\[
\begin{align*}
\text{Civ} + \text{Irr} + \text{Ind} &= 533,48 \\
0,00 &\leq \text{Civ} \leq 469,08 \\
41,66 &\leq \text{Irr} \leq 533,48 \\
1,42 &\leq \text{Ind} \leq 185,11
\end{align*}
\]  

(33).

Expressing the boundary values in percentage we obtain the following table.

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value [%]</th>
<th>Maximum value [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>0,0</td>
<td>87,9</td>
</tr>
<tr>
<td>Irrigational</td>
<td>7,8</td>
<td>100,0</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,3</td>
<td>34,7</td>
</tr>
</tbody>
</table>

Table 57. Minimum and maximum percentage values for the players
Min Priority – Scenario A

Hereafter, it is reported the graphic representation of the core (Figure 68) and the boundary values expressed in unit cost per cubic metre of water delivered (Table 58).

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value [€/mc]</th>
<th>Maximum value [€/mc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>0,000</td>
<td>0,111</td>
</tr>
<tr>
<td>Irrigational</td>
<td>0,004</td>
<td>0,050</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,003</td>
<td>0,349</td>
</tr>
</tbody>
</table>

Table 58. Minimum and maximum unit costs for the players
Min Priority – Scenario B
7.4. **Analysis of results**

We can observe that the evaluation of the minimum set of infrastructures and the relative management costs which determine the c.f. of the game, is dependent on the priority of use of resource and the infrastructures assigned to the players of a coalition.

In the research two different approaches have been analyzed. In the first case we have considered the maximum priority, i.e. the minimum set of infrastructures, used by a coalition, included firstly the works which are economically more convenient, avoiding those more onerous as for example the pumping. Instead, in the second approach, the users in coalition have been characterized by the minimum priority and are bound to use infrastructures and resource after their exploitation by possible players out of coalition.

The two priorities represent the extreme cases that can characterize a user in a water system, so their analysis can be very significant when analyzing cost function attribution.
For this reason, it is interesting to analyze the cost function related to the two approaches. Cost functions have been evaluated (see Figure 69) analyzing the same water system in which the irrigational user was eliminated to simplify the calculation, and we decided to calculate them considering the coalition formed only by civil user. The total water demand of the system has been valued fixed and equal to 220 Mm$^3$/year, then, varying the civil water demand from zero to the limit value of 220 Mm$^3$/year, we have obtained the cost function. In case of minimum priority, the marginality boundaries have been valued associating to the player out of coalition (in this case only the industrial user) a water demand complementary to that one of the civil user.

![Cost functions](image)

**Figure 69. Cost functions**

As was predictable, in case of low priority the cost function has a very high marginal value at the beginning, but both the curves arrive at the same point when the maximum water demand is reached. In case of high priority the cost function is increasing with an upward concavity: for small water demands it is necessary that few water works and the upper limit is represented by the activation of water drawing from the near basin through the pumping with a sensible increase of management costs. Analyzing the cost function with low priority, we note that in its final part the curve is decreasing: this behaviour is linked to the entity and typology of residual resources; in fact only in this zone the player can use a big amount of the most economical resources, reducing the use of the pumps. This is possible because, as we said, total water demand of the system has been assumed to be fixed.

The different trend of the two cost functions is better explained showing Figure 70; here the cheapest resource (coming from reservoirs) used yearly on average by civil user in function of its water demand is reported. In case of maximum priority
the player has a precedence of use on the whole water inputs, so for low entities of water demand this is completely satisfied by the stored resource: in this context the function is equal to the bisector of the quadrant. After a certain quantity of water request, the cheapest resource is insufficient, and so it is necessary to use the most onerous resource through the pumping and consequently the function is decreasing. Instead, in case of minimum priority, the stored volumes are not totally available for the user because it is influenced by preliminary use of the system by players out of coalition and it is necessary other resources for its satisfaction: therefore the curve is lower than the other. Moreover the function increases with a upward concavity: this trend is due to the fact that when the water demand of the user increases the complementary water demand of players out of coalition decreases: consequently there are more stored volumes available.

![Resource coming from reservoirs](image)

**Figure 70. Available resource coming from reservoirs**

It is also useful to examine the functions of unit cost per cubic metre of delivered water reported in Figure 71. In case of maximum priority the function is decreasing, until almost 120 Mm$^3$. Then, when it is necessary the use of onerous resources, such function is changed assuming an increasing trend. On the contrary, considering the minimum priority, the unit cost function is constantly decreasing.
Figure 71. Unit cost functions

In conclusion we can say that the trend of unit cost functions affects the c.f. of a game and consequentially it concerns the possibility to define the core. In fact, for decreasing functions, it is possible to state that the c.f. is subadditive and, therefore, the Grand Coalition is the most efficient solution for the system; while this assumption is not always valid when the functions have an increasing trend.

Those situations have been found in the previous applications. In fact we saw that in case of minimum priority, changing water demands, the c.f. of the game remained subadditive; instead for maximum priority, when it was reached an high level of exploitation of the resource, we obtained a non-subadditive c.f.
8. Application to the Flumendosa - Campidano water system

In the previous chapter we analyzed an exemplification of the Flumendosa – Campidano water system, in order to describe in details and in a more comprehensible way the different steps of cost allocation methodology.

The application to the complete system is described in the present chapter. Here it is exclusively considered the second methodological approach, with the assignment of the minimal priority using the resource by the user in the coalition. In this way a subadditive c.f. is surely obtainable and, consequently, we can surely define the core of the game.

8.1. Application to the complete system

Water system description

In Figure 72 we report the graph representing the complete Flumendosa – Campidano system, as previously described in Chapter 6. In the graph the water infrastructures and the centres of demand of the multi purpose schemes, that formed the water system, are present.
Figure 72. Flumendosa – Campidano water system
The different water infrastructures and their relative management costs, extracted from PSURI, are given in Table 59. Moreover, in the same table we report the values of maximum capacity for the reservoirs and the maximum flow for the pumping stations. To simplify the calculation we did not include upper bound for the diversion dams and for the transportation works. Some infrastructures do not present any management cost because, even if they are part of Flumendosa – Campidano, they do not belong to the multi purpose system, that is the object of this economic analysis. The cost linked to these infrastructures will be totally at their operators’ expense.

<table>
<thead>
<tr>
<th>Infrastructures</th>
<th>Code</th>
<th>Maximum limit of use</th>
<th>Maintenance cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dams</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Is Barrocus</td>
<td>Is Barrocus</td>
<td>12,24</td>
<td>0,267</td>
</tr>
<tr>
<td>Flumineddu</td>
<td>Flumineddu</td>
<td>1,94</td>
<td>0,280</td>
</tr>
<tr>
<td>Nuraghe Arrubiu</td>
<td>Nuraghe Arrubiu</td>
<td>300,00</td>
<td>0,524</td>
</tr>
<tr>
<td>Monte Su Rei</td>
<td>Monte su Rei</td>
<td>332,00</td>
<td>0,499</td>
</tr>
<tr>
<td>Sa Forada de S’Acqua</td>
<td>Sa Forada</td>
<td>1,41</td>
<td>0,531</td>
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<td>Genna Is Abis</td>
<td>25,41</td>
<td>0,510</td>
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<td>Bau Pressiu</td>
<td>8,50</td>
<td>0,382</td>
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<td>Simbirizzi</td>
<td>Simbirizzi</td>
<td>24,61</td>
<td>0,203</td>
</tr>
<tr>
<td>Bau Mela</td>
<td>Bau Mela</td>
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<td>Bau Mandara</td>
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</tr>
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<td>Bau Muggeris</td>
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<td>-</td>
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<td>Santa Lucia</td>
<td>3,70</td>
<td>-</td>
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<td>Corongiu</td>
<td>Corongiu</td>
<td>4,30</td>
<td>-</td>
</tr>
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<td><strong>Diversion dams</strong></td>
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<td>Ponte Maxia</td>
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</tr>
<tr>
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<td>S’Isca Rena</td>
<td></td>
<td>0,014</td>
</tr>
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<td>Casa Fiume</td>
<td></td>
<td>0,014</td>
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<tr>
<td>Monastir</td>
<td>Monastir</td>
<td></td>
<td>0,014</td>
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<tr>
<td>Rio Fanaris</td>
<td>Fanaris</td>
<td></td>
<td>0,014</td>
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<tr>
<td>Location</td>
<td>Station Code</td>
<td>Maximum Flow [Mm³/month]</td>
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<tr>
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<td>--------------</td>
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<tr>
<td>Santa Lucia</td>
<td>Santa Lucia</td>
<td>0,014</td>
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<td>Monti Nieddu</td>
<td>Monti Nieddu</td>
<td>0,014</td>
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<td><strong>Pumping stations</strong></td>
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<tr>
<td>Villanovatulo</td>
<td>7A.P1</td>
<td>0,66</td>
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<tr>
<td>Basso Flumendosa</td>
<td>7A.P6</td>
<td>12,96</td>
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<tr>
<td>Macchiareddu</td>
<td>7E.P3</td>
<td>10,37</td>
<td></td>
</tr>
<tr>
<td>Monti Nieuddu</td>
<td>7E.P6</td>
<td>1,24</td>
<td></td>
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<td>Sulcis</td>
<td>1A.P1</td>
<td>3,85</td>
<td></td>
</tr>
<tr>
<td>Sardara</td>
<td>2C.P3</td>
<td>5,18</td>
<td></td>
</tr>
<tr>
<td>Simbirizzi Irriguo</td>
<td>7B.P3_irr</td>
<td>15,56</td>
<td></td>
</tr>
<tr>
<td>Simbirizzi Civile</td>
<td>7B.P3_civ</td>
<td>6,22</td>
<td></td>
</tr>
<tr>
<td>Cixerri</td>
<td>7E.P1</td>
<td>10,37</td>
<td></td>
</tr>
<tr>
<td>Is Arenas</td>
<td>7B.P5</td>
<td>5,18</td>
<td></td>
</tr>
<tr>
<td><strong>Transportation Works</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Pipeline Sarcidano I</td>
<td>SARC1</td>
<td>0,048</td>
<td></td>
</tr>
<tr>
<td>Pipeline Sarcidano II</td>
<td>SARC2</td>
<td>0,091</td>
<td></td>
</tr>
<tr>
<td>Pipeline Sarcidano III</td>
<td>SARC3</td>
<td>0,065</td>
<td></td>
</tr>
<tr>
<td>Tunnel Flumineddu - Nuraghe Arrubiu</td>
<td>G1</td>
<td>0,082</td>
<td></td>
</tr>
<tr>
<td>Tunnel Nuraghe Arrubiu - Monte Su Rei</td>
<td>G2</td>
<td>0,064</td>
<td></td>
</tr>
<tr>
<td>Tunnel Monte Su Rei - Sarais</td>
<td>G3</td>
<td>0,117</td>
<td></td>
</tr>
<tr>
<td>Connection S’Isca Rena diversion dam</td>
<td>BF</td>
<td>0,090</td>
<td></td>
</tr>
<tr>
<td>Canal Adduttore</td>
<td>ADD</td>
<td>0,354</td>
<td></td>
</tr>
<tr>
<td>Tunnel Sa Forada - Casa Fiume</td>
<td>SF-CF</td>
<td>0,033</td>
<td></td>
</tr>
<tr>
<td>Aqueduct Mulargia - Cagliari I</td>
<td>ACQ1</td>
<td>0,245</td>
<td></td>
</tr>
<tr>
<td>Aqueduct Mulargia - Cagliari II</td>
<td>ACQ2</td>
<td>0,096</td>
<td></td>
</tr>
<tr>
<td>Aqueduct Mulargia - Cagliari III</td>
<td>ACQ3</td>
<td>0,389</td>
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</tr>
<tr>
<td>Aqueduct Mulargia - Cagliari IV</td>
<td>ACQ4</td>
<td>0,086</td>
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<td>Aqueduct Mulargia - Cagliari V</td>
<td>ACQ5</td>
<td>0,058</td>
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<tr>
<td>Connection Cixerri - Nuovo Partitore Sud-East</td>
<td>CIX-NSE</td>
<td>0,109</td>
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</tr>
<tr>
<td>Partitore Sud Est I</td>
<td>SE1a</td>
<td>0,204</td>
<td></td>
</tr>
<tr>
<td>Partitore Sud Est II</td>
<td>SE1b</td>
<td>0,203</td>
<td></td>
</tr>
<tr>
<td>Partitore Sud Est III</td>
<td>SE2</td>
<td>0,243</td>
<td></td>
</tr>
<tr>
<td>Partitore Sud Est IV</td>
<td>SE3</td>
<td>0.186</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>-----</td>
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<td></td>
</tr>
<tr>
<td>Nuovo Partitore Sud Est I</td>
<td>NSE1</td>
<td>0.219</td>
<td></td>
</tr>
<tr>
<td>Nuovo Partitore Sud Est II</td>
<td>NSE2</td>
<td>0.226</td>
<td></td>
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</tr>
</tbody>
</table>

**Table 59. Maintenance costs of infrastructures. Flumendosa – Campidano**

The pumping stations are also characterized by their energy cost; we chose to assign to it the same value used in the previous reduced scheme: 0.07 €/m³.

The nodes for water demands in the graph of Figure 72 are representative of the demand centres of the system, reported in Table 25 of Chapter 6. The monthly behaviour of the requests is the same previously adopted in Chapter 7 (see Table 26 and Figure 40).

The hydrological inputs of the system, that are linked to the reservoirs and to the diversion dams, have been taken from PSURi and they are shown below from
Figure 73 to Figure 76. These inputs have been inserted in the nodes of the graph called “Apo”. We remember that the regional document provides the series of runoffs on the basis of observed data.

The time horizon of analysis is always equal to 53 years, equivalent to the length of the hydrological series available from PSURI.
Figure 75. Hydrological input (c)

Figure 76. Hydrological input (d)
Cooperative game planning

In the CGT application the players of the game are the three macro users of the multipurpose system: civil, irrigational and industrial. These macro users are formed by all centres that are characterized by the same typology of water demand (as in Table 60).

To simplify the calculation and to group inside the same player the centres of demands with the same homogeneous characteristics of localization and management we have chosen to ignore the users of the scheme 6A. However, these are opportunistically considered during the optimization procedures.

<table>
<thead>
<tr>
<th>Player</th>
<th>Centre of demand</th>
<th>Total water request [Mm³/year]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>D41, D44, D45, D48, D51-SM, D51-SS, D51-Flut, D54, D57, D58, D58-Flut</td>
<td>98,70</td>
</tr>
<tr>
<td>Irrigational</td>
<td>D39, D42, D43, D46, D47, D49, D50-60, D52, D53, D59</td>
<td>81,99</td>
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<tr>
<td>Industrial</td>
<td>D55, D56</td>
<td>15,00</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>195,69</td>
</tr>
</tbody>
</table>

Table 60. Water request of the players. Flumendosa - Campidano

Since every player is formed by many agents, its management cost will be equal to the sum of the management costs of the infrastructures necessary to supply the demands belonging to it.

Optimization models set up

The adopted optimization procedures are the same described in the previous chapter: we generate a LP model using WARGI associated to every coalition. Analyzing the optimizations results we can individuate the infrastructures in service for every coalition and their relative management costs.

Marginality constraints definition

As we have previously described, when we assign the minimum priority to the players it is necessary to evaluate for every coalition their relative marginality constraints in order to determine the available resources and the residual capacity of the works. In this case the residual capacity is exclusively referred to the reservoirs and to the pumping stations because they are the only ones that have an upper bound related to the storage capacity transferring resources between time periods.
**Characteristic function valuation**

On the basis of the marginality constraints for every coalition, all the optimizations have been carried out and the relative set of used infrastructures have been identified. The results are reported in Table 61.

<table>
<thead>
<tr>
<th>Coalitions</th>
<th>Civil</th>
<th>Irrigational</th>
<th>Industrial</th>
<th>Civ +Ind</th>
<th>Irr +Ind</th>
<th>Civ +Irr</th>
<th>Grand</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

Table 61. Infrastructures used by coalition.
Flumendosa - Campidano

Hereafter we report for every coalition the total pumped volumes of every pumping station of the system during the whole time horizon of analysis.
From the previous tables it is clear that the Grand Coalition has not used all infrastructures. This means that under the current water requests and the hydrological inputs of PSURI the Flumendosa – Campidano water system is correctly sized and no pumping from the external source is required.

On the basis of the used infrastructures and the pumped volumes we obtain the following management costs related to the coalitions of the game.
Therefore the c.f. is the following.

<table>
<thead>
<tr>
<th>Coalition</th>
<th>Characteristic function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grand Coalition</td>
<td>293,36</td>
</tr>
<tr>
<td>Civil</td>
<td>277,78</td>
</tr>
<tr>
<td>Irrigational</td>
<td>220,02</td>
</tr>
<tr>
<td>Industrial</td>
<td>49,60</td>
</tr>
<tr>
<td>Civil – Irrigational</td>
<td>304,83</td>
</tr>
<tr>
<td>Irrigational - Industrial</td>
<td>206,54</td>
</tr>
<tr>
<td>Civil - Industrial</td>
<td>285,20</td>
</tr>
</tbody>
</table>

Table 64. Characteristic function. Flumendosa - Campidano

Finally we value the efficiency of the Grand Coalition comparing its cost with that one of the other alternatives.

<table>
<thead>
<tr>
<th>Combinations of Coalitions</th>
<th>Cost [M€]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grand Coalition</td>
<td>293,36</td>
</tr>
<tr>
<td>[Industrial] + [Civil – Irrigational]</td>
<td>354,43</td>
</tr>
<tr>
<td>[Civil] + [Industrial – Irrigational]</td>
<td>484,32</td>
</tr>
<tr>
<td>[Irrigational] + [Civil – Industrial]</td>
<td>505,22</td>
</tr>
<tr>
<td>[Irrigational] + [Civil] + [Industrial]</td>
<td>547,40</td>
</tr>
</tbody>
</table>

Table 65. Efficiency of the Grand Coalition. Flumendosa - Campidano

As it was expected, from an economic point of view the Grand Coalition is the most efficient combination for the system.

**Game solution**

According with the c.f. of the game the boundaries values of cost for every player are evaluated, see Table 66. As we said in the previous chapter the negative values are not admissible, therefore the minimum cost associable to the demands is equal to zero.
Analytically the core is represented by the set of allocations that verify the following system:

\[
\begin{align*}
\text{Civ} + \text{Irr} + \text{Ind} &= 293.36 \\
86.82 &\leq \text{Civ} \leq 277.78 \\
8.16 &\leq \text{Irr} \leq 220.02 \\
0.00 &\leq \text{Ind} \leq 49.60
\end{align*}
\]  

Hereafter in Figure 77 there is the graphic representation of the core.
The boundaries values expressed in percentage and in unit cost are reported in the following tables.

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>29,6%</td>
<td>94,7%</td>
</tr>
<tr>
<td>Irrigational</td>
<td>2,8%</td>
<td>75,0%</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,0%</td>
<td>16,9%</td>
</tr>
</tbody>
</table>

Table 67. Minimum and maximum percentage values for the players.
Flumendosa – Campidano

<table>
<thead>
<tr>
<th>Player</th>
<th>Minimum value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>0,017 €/mc</td>
<td>0,053 €/mc</td>
</tr>
<tr>
<td>Irrigational</td>
<td>0,002 €/mc</td>
<td>0,051 €/mc</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,000 €/mc</td>
<td>0,062 €/mc</td>
</tr>
</tbody>
</table>

Table 68. Minimum and maximum unit costs for the players.
Flumendosa – Campidano

8.2. Analysis of results

Starting from the obtained results it is possible to make some interesting analyses and comparison with the current allocation rates assigned by the Regional Board (ENAS).

First of all, we have to consider that the methodology has been applied to a limited part of the regional water system and it exclusively analyzes the management cost of the infrastructures. The study does not consider other typologies of cost, as the staff cost.

However, it is possible to extend the results to the whole multi purpose water system applying the percentage boundary values obtained to the total cost of ENAS. In this way we can also divide among the players some typologies of cost, as for example the staff costs, that are difficult to assign to the users.

Hereafter we report the budget of the ENAS expenses for the year 2010 (source Official Gazette of Sardinia of 03/07/2010).
The income ENAS for activities different from the sale of wholesale water are reported hereafter.

According to the previous tables, the income from rates necessary to reach the break-even are equal to the difference between the expenses and the income for activities different from the sale of wholesale water.

Now if we apply to this cost the percentage boundaries values, obtained for the Flumendosa – Campidano (see Table 67), we find the following values expressed in millions of euro per year.
Moreover, we can consider the volumes of wholesale water assigned for the year 2010 to the three multipurpose demands, reported in Official Gazette of Sardinia of 03/07/2010, to obtain a theoretical range of unit cost per cubic metre for every user (Table 74).

<table>
<thead>
<tr>
<th>User</th>
<th>Mm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>230,1</td>
</tr>
<tr>
<td>Irrigational</td>
<td>389,2</td>
</tr>
<tr>
<td>Industrial</td>
<td>30,4</td>
</tr>
<tr>
<td>Total</td>
<td>649,7</td>
</tr>
</tbody>
</table>

Table 73. Assigned wholesale water volumes for 2010

Such values are now comparable with the ENAS rates for 2010 on Table 75.

<table>
<thead>
<tr>
<th>User</th>
<th>Rate 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil I</td>
<td>0,025</td>
</tr>
<tr>
<td>Civil II</td>
<td>0,056</td>
</tr>
<tr>
<td>Irrigational I</td>
<td>0,005</td>
</tr>
<tr>
<td>Irrigational II</td>
<td>0,015</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,23</td>
</tr>
</tbody>
</table>
Comparing the values, we can note that for the civil and irrigational users their current rates are contained inside the theoretical range given by the CGT core (we ignore the little difference for the minimum civil value); while for the industrial it is outside. This fact implies that, under these conditions, the current rates do not belong to the core of the game and so they do not respect the illustrated CGT principles of equity and impartiality.

8.2.1. Hypothesis of rates

Moreover, according with preceding results, it is interesting to define a new rating plan based on allocation cost methodology. New rates have to be based on the theoretical range reported in Table 72 and they have to cover the ENAS expenses (Table 71), in order to respect the principle of the recovery of costs required from the European Directive 2000/60/EC. Therefore some rating hypotheses are proposed below.

Considering that the maximum theoretical value for the industrial demand is lower than its current rate, we decide to attribute to it its maximum theoretical limit equal to 0,1134 €/m$^3$. This value, on the basis of the assigned wholesale water volumes, determines a cover of the ENAS expenses equal to 16,9%.

The remaining 83,1% has to be divided between the civil and irrigational demands. If we choose to adopt the minimum theoretical value of 0,0015 €/m$^3$ for the irrigational demand, this covers only the 2,8% of the ENAS expenses. So the civil demand will be charged with the residual 80,3% which determines an unit cost equal to 0,0712€/m$^3$.

Another alternative is the one that applies to the irrigational demand its minimum current rate, i.e. 0,005 €/m$^3$, which determines a covering of expenses equal to 9,6%. The civil demand in this case will cover the residual 73,5% which determines an unit cost equal to 0,0652€/m$^3$.

One more hypothesis can be done associating the maximum current rate for the irrigational demand, equal to 0,015€/m$^3$ that cover the 28,6% of the expenses, and then to charge the civil demand with the residual 54,5%, that corresponds to a unit cost equal to 0,0483€/m$^3$.

<table>
<thead>
<tr>
<th>Player</th>
<th>Hypothesis 1 [€/mc]</th>
<th>Hypothesis 2 [€/mc]</th>
<th>Hypothesis 3 [€/mc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>0,0712</td>
<td>0,0652</td>
<td>0,0483</td>
</tr>
<tr>
<td>Irrigational</td>
<td>0,0015</td>
<td>0,0050</td>
<td>0,0150</td>
</tr>
<tr>
<td>Industrial</td>
<td>0,1134</td>
<td>0,1134</td>
<td>0,1134</td>
</tr>
</tbody>
</table>

Table 76. Hypothesis of rates.
To conclude this investigation, we finally executed an analysis of sensitivity of the rates between the civil and irrigational demands, fixing the value of industrial user equal to 0.1134 €/m³. We obtained that an increase of 0.01 €/m³ for the irrigational determines a decrease equal to 0.0169€/m³ for the civil, while the same increase applied to the civil determines a decrease for the irrigational equal to 0.0059€/m³.

<table>
<thead>
<tr>
<th>Player</th>
<th>[€/mc]</th>
<th>[€/mc]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil</td>
<td>+ 0,01</td>
<td>-0,0169</td>
</tr>
<tr>
<td>Irrigational</td>
<td>-0,0059</td>
<td>+0,01</td>
</tr>
</tbody>
</table>

Table 77. Analysis of sensitivity
9. Conclusions and perspective

9.1. Conclusions

In this thesis the problem of cost allocation in a water resources system has been faced and sorted out considering CGT. At the beginning it has emerged the necessity of a change in defining water rates; in particular, as we saw in Chapter 2, the Directive 2000/60/EC requires the realization of an economic analysis of the water use in a river basin-scale; the Directive also provides that the pricing politics have to take into account the principle of recovery of costs and the economic sustainability of the final user. Moreover we showed how the currently methods of allocation do not take into account the user's willingness to pay and, frequently, such methods do not permit the total costs covering.

Moreover we have underlined how the CGT can provide a valid contribution to the above mentioned problems. As we have widely described in Chapter 4, the CGT provides the necessary instruments to analyze those situations in which is fundamental to search a sharing mechanism that should be efficient, fair and it has to provide the appropriate incentives among the involved parts. Therefore, the cost allocation problem in a water system has been valued as a game in which it is necessary to determine the right payoff to be assigned to the different players, which represent in our case the water users. By the use of the costs allocation methods of CGT it has been possible to “make explicit” the process of negotiation with mathematical formulas which implement properties that guarantee equity, fairness and cooperation among the players.

For these reasons, we have developed a methodology which permits to allocate the water services cost by the use of CGT. Such a methodology permits to achieve
a cost sharing which is acceptable, provides an adequate justification of the adopted criteria and also favours the cooperation among the users in order to maximize the efficiency of the management of the system.

For that, we realized a cooperative game in which the users of a water system act as players in a coalition to achieve their water supply at the minimum cost. The use of the optimization model WARGI allows to value the least cost for every coalition, defining the optimal set of infrastructures necessary to satisfy the water requests of the players. WARGI permits to represent any water system and to easily enter the relative characteristic data: this makes the methodology totally applicable to every water system. From this point of view, such methodology does not present any macroscopic limit, except the one linked to the number of players of the game because, as we can noted from the research, this element influences the number of optimizing operations to be realized.

The minimum cost of the coalitions of the game defines the characteristic function: this is the key element of the methodology, through which is possible to get the solution of the game. We chose to use the core solution, that represents the area of the admissible cost allocation values and, through this, it is possible to supply the boundaries values of cost for every player. Inside the core, as we saw, there are all the allocations which satisfy the principles of equity, fairness, justice, efficiency and which guarantee the costs recovery. The core of a cooperative game can represent an useful instrument to define or to adjust water rates. The valuation of the core is a valid support in the management of the water resources for the achievement of the economic analysis required by the European Directive.

The application to the water system Flumendosa-Campidano in Sardinia has permitted to better appreciate the most important characteristics of the methodology and to analyze some critical points. In this specific case, we have examined the maintenance costs added to the pumping costs and then we got on to their allocation among the three principal water users, defining their relative range of costs.

We have also considered two different approaches, associating to the players the minimum and the maximum priority of the use of resource and infrastructures of the system. The obtained results have permitted to weigh the pros and the limitations of the methodology and also they have enabled to value the evolution of the allocation in functions of the assigned priority.

To conclude we want to underline not the computations and the numerical results presented in the thesis, but the procedure adopted to achieve them. In fact, such methodology is easily adaptable to allocate different typologies of cost and it is applicable to every water system: the only need is to use an adequate optimization model to calculate the minimum cost linked to the coalitions.
9.1.1. Original contributions

This thesis has provided an original contribution both in mathematical science, in particular in Game Theory, and in hydro-economic modelling, especially in water resources management.

The main novelty is represented by the definition of a methodology that implements the concepts and the methods of CGT for water resources systems. During this research, other contributions have come up and they are listed hereafter:

- the definition of a criterion for the allocation of management costs in a multi purpose water system;
- the calculation of the characteristic function in a cooperative game representing a complex water system through the use of an optimization model;
- the assignation of water services costs to the water users in function of their use of infrastructures;
- the calculation of the range of costs associated to a water user;
- the analysis of the relationships between the cost functions and the characteristic function in a cooperative game;
- the use of a methodology of cost allocation which makes explicit, clear and comprehensible the applied methods, with a transparent decision-making process;
- the development of a support that realizes the economic analysis of water use in a river basin-scale, as required by the European Directive;
- the definition of a method which favours the cooperation of the water users and reduces rivalry and disputes linked to the water resource;
- the realization of an instrument for the estimation of water rates which:
  - consider the user’s willingness to pay;
  - respect the cost recovery principle stated by European Directive;
  - are equal, impartial and commonly shareable;
  - are justified by a clear and comprehensible method of calculation, which permits to show to the user the motivations on the basis of a determined water price;
  - are valued in function of an hydrology analysis with a long time-series.
9.2. Future research

At the end of this document, on the basis of the achieved results and the limitations found in the development of the work, the following main possible developments of research have been individuated:

- to study in details the cost functions of the infrastructures in order to improve the information to be entered in the optimization model and the quality of the economic results;

- to extend the application of the methodology to the whole regional multi purpose water system and to the downstream water systems, as inside the civil districts or the irrigational areas;

- to expand the number of players considering other water users, as the hydroelectric production, the fish farming or the recreational uses;

- to develop an optimization tool which automates the calculation process of the coalition costs;

- to match the optimizing process with a pre and/or post simulative phase in order to value different scenarios with different management alternatives;

- to explore the possibilities offered by the application of the CGT to the systems characterized by insufficient resources for the total supply of the users. With regards to this, we suggest the use of a particular class of games called bankruptcy games (Curie et al., 1987): these are particularly adapted for the systems marked out by scarce water resources;

- to examine different levels of priority associated to the water users. In this case it will be necessary to decide preliminarily which level has to be assigned to the coalitions formed by players with different priorities. For example it will be possible to choose the lowest priority of the players which belong to the coalition;

- to represent the players not only in function of the typology of water user (civil, irrigational, industrial) but also in function of a specific level of their water request. For example we can divide the civil demand in two players: one related to the 80% of the total request, and the other to the remaining 20%.

Particularly, these two last aspects seems to be very interesting, because put together they can represent more adherently the current management behaviour carried out in water systems by water Authorities. In fact, they administrate the resource mainly on the basis of different priorities linked to the typology of the users and to their level of flexibility.


MIUR (2007) Azioni Integrate Italia Spagna. Sistema di supporto alle decisioni per la definizione del programma di misure per il raggiungimento degli obiettivi ambientali ed economici previsti dalla Direttiva Europea Quadro 2000/60 sulle acque. Ministro dell’Istruzione, dell’Università e della Ricerca, Italy


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Appendix A - Introduzione

Motivazione della ricerca

La gestione dei servizi idrici

L’acqua, l’elemento essenziale per la vita e fattore indispensabile per lo sviluppo umano, oltre che essere considerato da sempre come bene sociale, ha assunto negli ultimi anni anche la caratteristica di bene economico. Di conseguenza risulta importante dare alla risorsa idrica il giusto valore che tenga conto del suo utilizzo, consumo, deterioramento e dell’eventuale inquinamento, in maniera da incoraggiare l’utente ad un uso più sostenibile.

In Italia e nella maggior parte dei paesi europei l’assegnazione di sovvenzioni, sussidi, rimborsi e contributi da parte dei governi alle varie aziende, consorzi ed enti gestori del servizio idrico è risultata per anni, ed è tuttora, una pratica ampiamente diffusa, quasi dovuta, che ha portato il più delle volte ad un deprezzamento della risorsa.

Tuttavia sono in fase di attuazione nuove politiche economiche comunitarie più restrittive; in particolare, la recente Direttiva Europea 2000/60/CE (EU, 2000) ha promosso la necessità di un’analisi economica dell’uso idrico, introducendo il principio del recupero dei costi, con l’obiettivo di sostenere la salvaguardia e il miglioramento qualitativo e quantitativo dei corpi idrici.

In questo contesto risulta fondamentale portare avanti una politica tariffaria che sia solidamente fondata su un’attenta analisi di allocazione dei costi tra gli utenti di un sistema di risorse idriche.
La Direttiva Quadro 2000/60/CE

Come detto in precedenza, negli ultimi anni la Comunità Europea ha posto particolare attenzione alle problematiche in materia di risorse idriche; in tal contesto si inserisce la Direttiva 2000/60/CE che si pone l’obiettivo di perseguire il miglioramento dello stato dell’ambiente e in particolare dei corpi idrici ricadenti all’interno del territorio comunitario.

La Direttiva stabilisce un quadro di riferimento nella politica dell’acqua ed ha come obiettivo principale quello di raggiungere il buono stato di salute della risorsa idrica, proteggendola ed evitando il suo deterioramento per gli usi futuri. Per ottenere questo obiettivo viene assegnato un peso molto importante all’analisi idrologica finalizzata alla quantificazione della risorsa, alla gestione delle risorse idriche, alla partecipazione, informazione e consultazione pubblica ed infine all’analisi economica per garantire l’efficienza dei sistemi di utilizzazione.

Le problematiche legate alla definizione dei criteri per la gestione economicamente efficiente dei sistemi idrici, rappresenta uno dei temi più importanti presenti nella Direttiva Europea. Questi aspetti sono trattati in particolare negli articoli 5 e 9 della stessa. Nell’art. 5 è previsto che gli Stati membri provvedano affinché, per ciascun distretto idrografico, […] sia effettuata […] e completata entro quattro anni dall’entrata in vigore della Direttiva […] un’analisi economica dell’utilizzo idrico. Nell’art. 9 è detto che gli Stati membri tengono conto del principio del recupero dei costi dei servizi idrici, compresi i costi ambientali e relativi alle risorse, prendendo in considerazione l’analisi economica effettuata […] in particolare, secondo il principio di «chi inquina paga». Inoltre, entro il 2010, si dovrà provvedere affinché le politiche dei prezzi dell’acqua incentivino adeguatamente gli utenti a usare le risorse idriche in modo efficiente […] e si pervenga al recupero dei costi dei servizi idrici a carico dei vari settori di impiego dell’acqua, suddivisi almeno in industria, famiglie e agricoltura, sulla base dell’analisi economica effettuata secondo l’allegato III e tenendo conto del principio «chi inquina paga» (EU, 2000).

L’introduzione degli indicatori economici sopracitati, necessari per il raggiungimento degli obiettivi ambientali, implica riconoscere alla risorsa idrica un valore economico, proprio delle risorse finite. L’analisi economica riveste quindi un ruolo decisivo nella gestione delle risorse idriche e nel disegno delle nuove politiche dei prezzi dell’acqua.

Il problema dell’allocazione dei costi e la Teoria dei Giochi Cooperativi

I metodi tariffari attualmente utilizzati, che verranno analizzati nel dettaglio nel Capitolo 6, hanno il difetto di ignorare il problema della motivazione: ci si chiede, infatti, perché gli utenti finali debbano accettare un’assegnazione che ecceda i loro costi opportunità o la loro disponibilità a pagare (Young, 1985).
Il problema principale per la definizione di una nuova politica tariffaria, quindi, non risiede nella ricerca di una modalità di ripartizione dei costi tra gli utenti, ovvero nella ricerca di una determinata legge di allocazione delle spese, bensì su come ripartire in maniera equa e giusta i costi sostenuti. Ciò significa trovare un’allocazione dei costi che sia imparziale per tutti i partecipanti al progetto al fine di promuovere e garantire la collaborazione fra gli utenti e quindi la fattibilità di un progetto comune che permetta la riduzione dei costi per tutti i beneficiari.


La Teoria dei Giochi sviluppata intorno alla metà dello scorso secolo (Von Neumann & Morgenstern, 1944) analizza situazioni di conflitto in vari ambiti e ne ricerca soluzioni competitive e/o cooperative, ovvero studio le decisioni individuali in situazioni in cui vi sono interazioni tra diversi soggetti decisionali. Nella letteratura scientifica sono presenti numerosi casi di ripartizione dei costi che utilizzano i principi della CGT: gli ambiti applicativi sono vari e concernono anche studi legati alle risorse idriche (TVA, 1938; Young & Okada, 1982; Lippai & Heaney, 2000; Deidda et al., 2009).

Attraverso l’uso delle tecniche di assegnazione dei costi proprie della CGT è possibile “esplicitare” il processo di negoziazione attraverso formule matematiche che implementano proprietà tali da garantire l’equità, la giustizia e la cooperazione tra gli utenti coinvolti in un progetto, con il fine di ottenere una soluzione accettabile da tutti.

Tuttavia, l’applicazione della CGT si è limitata essenzialmente all’ambito economico e matematico, evitando la complessità e l’eterogeneità dei problemi di carattere ingegneristico, come nel caso della gestione dei sistemi di risorse idriche. Il calcolo di un gioco cooperativo richiede un’analisi del costo minimo del sistema, il che implica un processo di ottimizzazione il cui ordine di grandezza cresce esponenzialmente in funzione del numero di agenti coinvolti. Pertanto la necessità di disporre di adeguati strumenti di modellazione è risultato il principale ostacolo per la risoluzione di problemi di assegnazione del costo nel caso di sistemi complessi (Deidda, 2009).
Obiettivi della ricerca

L’obiettivo principale del presente lavoro di ricerca è lo sviluppo di una metodologia basata sull’applicazione della CGT che contribuisca al processo di definizione di tariffe idriche conformi ai principi della Direttiva Europea 2000/60/CE.

La metodologia risulta di carattere generale, adattabile alle condizioni di differenti bacini idrografici e compatibile con gli strumenti in uso per la modellazione dei sistemi di risorse idriche. Il suo ambito di applicazione si potrà estendere alla soluzione di problemi di assegnazione dei costi sia a livello locale che a livello regionale.

Con una metodologia di allocazione basata sulla CGT si potrà ottenere una ripartizione dei costi tra gli utenti che sia condivisibile, che fornisca un’adeguata giustificazione dei criteri adottati e che favorisca la cooperazione tra i soggetti interessati al fine di massimizzare l’efficienza nella gestione della risorsa idrica, obiettivo di notevole importanza nell’ambito dei sistemi idrici mediterranei caratterizzati da fenomeni di carenza idrica.

Il presente lavoro di tesi si propone pertanto di fornire un contributo originale sia nell’ambito delle scienze matematiche sia in quello della modellazione idro-economica.

Infine si fa presente che tale ricerca si inserisce all’interno del progetto internazionale Azioni Integrate Italia-Spagna (MIUR, 2007) che prevede la collaborazione tra il Dipartimento di Ingegneria del Territorio dell’Università di Cagliari e il Departamento de Ingeniería Hidráulica y Medio Ambiente della Universidad Politécnica de Valencia, con l’obiettivo di sviluppare un sistema di supporto alle decisioni per la definizione del programma di misure per il raggiungimento degli obiettivi ambientali ed economici previsti dalla Direttiva Europea 2000/60.

Metodologia di allocazione dei costi

L’applicazione della metodologia prevede inizialmente l’identificazione di tutti gli aspetti necessari alla descrizione e caratterizzazione del sistema idrico di studio; successivamente occorre impostare il gioco cooperativo definendo i giocatori e la tipologia dei costi che si vuole ripartire. I giocatori possono rappresentare sia singole utenze che loro raggruppamenti funzionalmente coerenti, come nel caso di utenze appartenenti ad un’unica macrodomanda (ad esempio un consorzio irriguo o industriale, complessi urbani, ecc.).

Si passa quindi alla fase più importante, ovvero la definizione della funzione caratteristica del gioco, elemento base della CGT. Questa è costituita dall’insieme dei valori di costo minimo associati a tutte le possibili coalizioni di giocatori, la cui
valutazione si effettua attraverso il programma WARGI (Sechi & Zuddas, 2000; Manca et al., 2004; Sechi & Sulis, 2009) basato su un modello di ottimizzazione specificatamente sviluppato per sistemi di risorse idriche. Il programma permette di costruire in modalità di grafica interattiva il sistema idrico di studio e di inserire agevolmente i dati richiesti (economici, idrologici, idraulici, infrastrutturali, ecc.) per la definizione funzionale del sistema.

Una volta definita la funzione caratteristica è possibile fornire la soluzione del gioco applicando i metodi di ripartizione dei costi propri della CGT.

**Il sistema idrico di studio**

La metodologia è stata validata applicandola allo schema idrico Flumendosa – Campidano situato in Sardegna.

L’isola è ubicata al centro del bacino occidentale del Mediterraneo e si estende per una superficie di circa 24 mila km² con una popolazione di 1.648.000 abitanti. Il clima è prettamente mediterraneo, caratterizzato da un lungo periodo di siccità estiva e da inverni miti e piovosi con gelate sporadiche.


Le infrastrutture appartenenti al sistema regionale multisettoriale sono state accorpati in diversi “schemi” in relazione all’uso della risorsa, attribuendo al medesimo schema tutte le opere idrauliche che, se pur non direttamente interconnesse tra loro, concorrono al soddisfacimento dei fabbisogni idrici del medesimo territorio.

Il sistema Flumendosa – Campidano risulta costituito da più schemi multisettoriali e permette l’approvvigionamento delle utenze della zona centro meridionale dell’isola.
**Struttura del documento**

Il documento si struttura in nove capitoli, incluso il primo capitolo introduttivo.

Nel Capitolo 2 viene descritta la normativa in tema di gestione di risorse idriche a livello europeo, nazionale e regionale, presentando nel dettaglio le norme di riferimento utilizzate nella ricerca.

Nel Capitolo 3 viene analizzato il problema dell’allocazione dei costi, revisionando i principali metodi di ripartizione adottati, introducendo il concetto di “willingness to pay” e fornendo una possibile risoluzione al problema introdotto.

Il Capitolo 4 è dedicato alla presentazione della CGT, con una breve descrizione iniziale della più generale Teoria dei Giochi. Si espongono nel dettaglio le definizioni, i principi e le soluzioni della CGT e si illustrano in conclusione alcune applicazioni a sistemi complessi.

Il modello di ottimizzazione WARGI viene descritto nel Capitolo 5. Vengono analizzate nel dettaglio le fasi necessarie per l’ottimizzazione di un sistema idrico e si illustrano le modifiche apportate al programma per meglio adattarlo all’esigenze del lavoro di ricerca.

Nel Capitolo 6 si descrive il sistema idrico di studio, il Flumendosa – Campidano, con le relative infrastrutture idrauliche e utenze collegate. Inoltre, si analizza in generale il sistema idrico della Sardegna e i vari settori idrici che lo compongono.

Il Capitolo 7 è dedicato alla descrizione della metodologia di allocazione dei costi dei servizi idrici basata sull’uso delle tecniche della CGT. Nel capitolo vengono presentati i risultati ottenuti su un’esemplificazione dello schema Flumendosa – Campidano analizzando due scenari di richiesta e due differenti approcci metodologici.

L’applicazione della metodologia allo schema completo Flumendosa – Campidano viene realizzata nel Capitolo 8 nel quale si propone in conclusione l’analisi dei risultati ottenuti e l’ipotesi di un nuovo assetto tariffario per il sistema idrico.

La tesi si conclude con il Capitolo 9 nel quale vengono illustrati alcuni possibili sviluppi futuri di ricerca e dove vengono presentate le conclusioni e i contributi originali forniti dal lavoro.

Sono inoltre presenti due appendici finali nelle quali si riportano in lingua italiana il capitolo introduttivo e quello conclusivo.
Appendix B – Conclusioni e linee future di ricerca

Conclusioni

Nel presente lavoro di tesi è stato affrontato il problema dell’allocazione dei costi in un sistema di risorse idriche utilizzando le tecniche della CGT. È emerso come risulti necessario un cambiamento nelle politiche di definizione delle tariffe idriche. In particolare, come abbiamo visto nel capitolo 2, la Direttiva Europea 2000/60/CE richiede che venga effettuata a scala di bacino un’analisi economica dell’uso idrico e che le politiche dei prezzi tengano conto del principio del recupero dei costi e della sostenibilità economica da parte dell’utente finale. Inoltre abbiamo mostrato come i metodi di allocazione attualmente utilizzati non tengano conto della disponibilità a pagare dell’utente e come il più delle volte non permettano la totale copertura dei costi.

Abbiamo quindi evidenziato come la CGT riesca a fornire un valido contributo nell’ambito delle problematiche sopraelencate. Come ampiamente descritto nel capitolo 4, la CGT fornisce gli strumenti necessari per analizzare situazioni in cui risulta basilare la ricerca di un meccanismo di ripartizione che sia efficiente, giusto e fornisca appropriati incentivi tra le varie parti coinvolte. Il problema dell’allocazione dei costi in un sistema idrico è stato, quindi, visto come un gioco in cui occorre determinare il giusto pay-off da assegnare ai diversi giocatori, nel nostro caso rappresentanti gli utenti idrici. Attraverso l’uso delle tecniche di assegnazione dei costi proprie della CGT è stato possibile “esplicitare” il processo di negoziazione attraverso formule matematiche che implementano proprietà tali da garantire l’equità, la giustizia e la cooperazione tra gli utenti coinvolti in un progetto, con il fine di ottenere una soluzione accettabile da tutti.
È stata quindi definita una metodologia che permette di effettuare l’allocazione dei costi dei servizi idrici sfruttando la CGT e i modelli di ottimizzazione dedicati alla gestione delle risorse idriche. Questa permette di ottenere una ripartizione che sia condivisibile, che fornisca un’adeguata giustificazione dei criteri adottati e che favorisca la cooperazione tra i soggetti interessati al fine di massimizzare l’efficienza della gestione del sistema.

Si è pertanto realizzato un gioco cooperativo nel quale gli utenti di un sistema idrico agiscono come giocatori in coalizione per il raggiungimento del loro approvvigionamento idrico al minimo costo. L’uso del modello di ottimizzazione WARGI permette di valutare il costo minimo per ogni coalizione, definendo il set ottimale di infrastrutture necessario per il soddisfacimento delle richieste idriche dei giocatori. L’utilizzo del modello di ottimizzazione WARGI permette di rappresentare qualsiasi sistema idrico e di inserire agevolmente i relativi dati caratteristici: ciò rende la metodologia totalmente applicabile a qualunque sistema di risorse idriche. Da questo punto di vista la metodologia non presenta alcuna limitazione macroscopica, se non quella legata al numero di giocatori scelti, poiché emerge dalla ricerca, esso influenza il numero di operazioni di ottimizzazione da realizzare.

L’insieme dei costi minimi del gioco definisce la funzione caratteristica, elemento chiave della metodologia, sulla base della quale si procede alla definizione della soluzione del gioco. Si è scelto di utilizzare la soluzione insiemistica del nucleo, che rappresenta l’area delle allocazioni di costo ammissibili e tramite esso è possibile fornire i limiti di costo associabili ad ogni giocatore. Al suo interno, come abbiamo visto, sono presenti tutte le allocazioni che soddisfano i principi di equità, accettabilità, giustizia, efficienza e che garantiscono la totale copertura dei costi del sistema. Il nucleo di un gioco cooperativo può rappresentare un utile strumento nella definizione o nell’adeguamento delle tariffe idriche che soddisfino i principi sopracitati. La valutazione del nucleo risulta inoltre un valido supporto nell’ambito della gestione delle risorse idriche per il compimento delle analisi economiche richieste dalla Direttiva Europea.

L’applicazione al sistema idrico Flumendosa – Campidano in Sardegna ha permesso di apprezzare maggiormente le caratteristiche più importanti della metodologia. Nel caso in esame sono stati analizzati i costi di manutenzione del sistema sommati a quelli legati all’utilizzo degli impianti di sollevamento e si è proceduto alla loro allocazione fra i tre principali utenti idrici, definendo il relativo range di costo associato.

Sono stati considerati due differenti approcci associando ai giocatori la priorità massima e minima di utilizzo della risorsa e delle infrastrutture del sistema. I risultati ottenuti hanno permesso di analizzare i pregi e i limiti della metodologia e hanno permesso di valutare l’evoluzione della ripartizione del costo associato agli utenti al modificare della priorità assegnata.

In conclusione vogliamo focalizzare l’attenzione non tanto sui risultati numerici ottenuti quanto alla procedura realizzata per il loro conseguimento. La
metodologia infatti è facilmente adattabile per l’allocazione di differenti tipologie di costo e risulta applicabile a qualunque sistema idrico, con l’unica necessità di disporre di un adeguato modello di ottimizzazione per il calcolo del costo minimo legato alle coalizioni.

**Contributi originali**

Il presente lavoro di tesi ha permesso di fornire un contributo originale sia nell’ambito delle scienze matematiche, in particolare della Teoria dei Giochi, sia in quello della modellazione idro-economica per la gestione dei sistemi di risorse idriche.

La principale novità risiede nella definizione di una metodologia per l’implementazione dei concetti e dei metodi caratteristici della CGT per i sistemi di risorse idriche con l’utilizzo delle tecniche di ottimizzazione.

Durante il lavoro di ricerca sono emersi anche ulteriori contributi che elenchiamo di seguito:

- la definizione di un criterio per la ripartizione dei costi di gestione di un sistema idrico multissettoriale;
- il calcolo della c.f. in un gioco cooperativo rappresentante un sistema idrico complesso;
- l’assegnazione dei costi dei servizi idrici in funzione dell’utilizzo delle infrastrutture da parte degli utenti;
- il calcolo del range di ammissibilità dei costi associati ad un utente di un sistema idrico;
- l’analisi delle relazioni tra funzioni di costo e c.f. in un gioco cooperativo;
- l’utilizzo di una metodologia di ripartizione dei costi che rende espliciti, chiari e comprensibili i metodi applicati;
- lo sviluppo di un supporto per la realizzazione dell’analisi economica a scala di bacino dell’uso idrico, come richiesto dalla Direttiva Quadro;
- la definizione di un metodo che promuove la cooperazione degli utenti in sistema idrico, riducendo la rivalità e le dispute legate alla risorsa;
- l’attuazione di uno strumento di valutazione di tariffe idriche che:
  - considerino la disponibilità a pagare da parte dell’utente;
  - rispettino il principio del recupero dei costi della Direttiva Europea;
  - siano eque, imparziali e condivise da tutti;
  - siano giustificate da un metodo di calcolo chiaro e comprensibile, che permette di mostrare all’utente le motivazioni alla base di un determinato costo associato;
  - siano valutate in funzione di un analisi idrologica su un ampio intervallo temporale di studio.
Sviluppi futuri

A conclusione del presente documento, sulla base dei risultati ottenuti e delle limitazioni incontrate nello sviluppo della tesi, sono stati individuati i seguenti sviluppi futuri di ricerca:

- approfondire lo studio delle funzioni di costo delle infrastrutture per migliorare le informazioni da inserire nel modello di ottimizzazione e la qualità globale dei risultati economici;
- estendere l’applicazione della metodologia all’intero sistema idrico multisettoriale regionale e a sistemi idrici secondari, come i distretti civili o le reti irrigue interne ai consorzi di bonifica;
- ampliare il numero dei giocatori considerando altri usi idrici, come la produzione idroelettrica, l’itticoltura o gli usi ricreativi;
- sviluppare un modello di ottimizzazione che automatizzi il processo di calcolo del costo delle coalizioni;
- abbinare al processo di ottimizzazione una fase pre e/o post simulativa al fine di valutare differenti scenari con varie alternative gestionali;
- esplorare le possibilità offerte dall’applicazione della CGT a sistemi caratterizzati da risorse idriche insufficienti per il completo approvvigionamento degli utenti. Si suggerisce a riguardo l’utilizzo di una particolare classe di giochi denominati di bancarotta (Fragnelli et al., 2008) che risultano particolarmente adatti per sistemi caratterizzati da risorse scarse;
- esaminare differenti livelli di priorità associati alle utenze. In tal caso occorrerà decidere preliminarmente quale livello assegnare alle coalizioni formate da giocatori aventi differenti priorità, ad esempio si potrà scegliere di utilizzare la più bassa priorità dei giocatori appartenenti ad esse;
- considerare i giocatori secondo i livelli di consumo delle utenze idriche. In tal caso ogni giocatore rappresenterà una tipologia di domanda e una sua determinata percentuale di richiesta idrica.

Quest’ultimi due aspetti risultano particolarmente interessanti, in quanto rispecchiano l’effettivo comportamento gestionale attuato nei sistemi idrici da parte delle autorità competenti. Infatti queste gestiscono il sistema sulla base di differenti priorità legate alla tipologia delle utenze e ai loro livelli di consumo.