NEW OPTIMIZATION MODELS FOR EMPTY CONTAINER MANAGEMENT

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Gennaio 2007
ACKNOWLEDGEMENTS

My first thanks go to Prof. Alessandro Olivo and Prof. Paola Zuddas, who gave me the opportunity of attending a doctoral programme and supervised this thesis. I am also sincerely grateful to them for providing me with confidence and freedom in developing the research programme.

I am completely in debt to Prof. Teodor Gabriel Crainic. He not only provided valuable comments, that helped me to improve the quality of this thesis, but also motivated my studies by his friendliness. I would also like to thank him for accepting to be a member of the doctoral committee.

I would like to take this opportunity to express my gratitude to my colleague Antonio Manca for his effort in answering all my boring questions on several computer programming issues.

I want to thank some industrial experts like Antonio Musso of Grendi Trasporti Marittimi, Ivano Bruzzone, Alessandra Lampugnani and the nice people of CMA-CGM Italy for having devoted their valuable time to this research programme.

Finally, I feel the need to express my sincere gratitude to my family and my girlfriend Laura. This thesis would not have been possible without their love. Last but not least, special thanks go to my friends, because of their support during these invaluable years of doctoral studies. It is impossible to mention them all as there are too many to mention!
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Chapter 1

Introduction

1.1 Intermodal container-based transportation

Freight transportation is a relevant activity for social, industrial and commercial processes. Recent decades have seen meaningful changes in cargo handling techniques, amongst which one the most important has been the introduction of containers. Their success mainly depends on the reduction of an almost infinite number of shapes and dimensions of goods to a small set of standardized units. Such equipment results in higher productivity during handling phases and provides undeniable advantages in terms of security, losses and damages.

Containers have transformed carrier activity from a port-to-port to a door-to-door service over different transportation modes, because transshipment can be performed easily. In a widely accepted meaning, intermodal freight transportation is the movement of goods in one and the same loading unit (for instance a container) which uses successive and various transportation modes without handling of goods during the transfer between modes (European Conference of Ministers of Transport, 1993). Therefore Intermodal Transportation can be seen as a multi-modal network of container transportation services, connecting the initial shipper to the final recipient. Such transportation modes are usually offered by different carriers and are coordinated at various intermodal terminals. Several activities (loading, unloading, storage, etc.) are performed to ensure door-to-door transportation for customers. A typical example in this field consists of a container moving by truck to a port or a rail yard connected to a port. Then one or more vessels move the container to the final port. Finally a combination of modes (truck, rail, barge, etc.) ensures goods to be delivered to the final destination (Macharis and Bontekoning, 2004).

Many data witness the remarkable development of container based transportation. According to TBridge S.p.A (http://www.tbridge.it), an Italian society skilled in consulting, while 50 millions TEU were moved in 1980, the increase of worldwide trade yielded 400 millions TEU hauled in 2005. Moreover the trend of container-based traffic
indicates an average growth of 9.2% in the next future. In 2005, 35 millions TEU were handled by Mediterranean ports and, if that tendency were to persist, this number would exceed 60 million TEU in 2010. Moving the focus onto transportation capacity, in 2005 the worldwide fleet was made up of more than 7.3 million TEU and forecasts indicate an expected increase of 11% in the next few years.

The effectiveness of container transportation has yielded significant consequences. Due to globalization of world economies and the evolution of regulatory environment, it has fueled international trade and many firms have set up productive activities with sales bases all over the world. Nowadays containerships allow finished or seed-ended products to be produced in the most suitable place and hauled safely where they will be consumed or used in production processes.

The effects of container-based transportation have been relevant on intermodal terminals as well. Ports have been deeply modified to accommodate such cellular vessels by investing in expensive berthing equipment, deepening and widening their channels. The efficiency of such facilities represents a vital component for the performance of the whole transportation network. Several problems and mathematical models for terminal container management have been presented. Many of them are described in Crainic and Kim (2004) and Moccia (2005).

The large part of the worldwide containerized fleet is owned by shipping companies, which are organized in hub-and-spoke networks. Their huge vessels, due to uneconomical convenience to stop often and the impossibility to berth in the majority of ports, operate between a restricted number of transshipment terminals (hubs). Then smaller vessels connect the hubs with origin/destination ports (spokes). It is worth noting that all intermodal transportation services (for example postal services and air transportation) adopt a hub and spoke organization, as it allows a more efficient usage of available resources. Nonetheless the consolidation of flows among hubs enhances the need of such arrangement within the domain of container-based transportation to cut down costs. According to TBridge S.p.A (http://www.tbridge.it), while the operating cost per slot for a 2000 TEU capacity vessel is 650$, it may be estimated to 525$ for a 7000 TEU fully cellular containership. This cost takes into account manning, repair and maintenance, insurance, lubes, fuel and port charges. The tendency towards larger vessels is expected to continue in the future. At the beginning of March 2006 the order
book of shipping companies indicates 160 vessels carrying over 7500 TEU and some 10000 TEU vessels have already been ordered.

The need to achieve scale economies and controlling the trend of transportations fees has yielded several mergers resulting in an oligopolistic shape of the market, where a limited number of shipping companies shares the world traffic. Nowadays more than 450 carriers provide maritime transportation and the capacity of the top four accounts for the 40% of total capacity.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Operator</th>
<th>TEU</th>
<th>Vessels</th>
<th>% of total fleet capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maersk-SeaLand + P&amp;O</td>
<td>1673691</td>
<td>588</td>
<td>19,10%</td>
</tr>
<tr>
<td></td>
<td>Nedlloyd</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Mediterranean Sh. Co</td>
<td>807374</td>
<td>281</td>
<td>9,20%</td>
</tr>
<tr>
<td>3</td>
<td>CMA-CGM Group</td>
<td>517369</td>
<td>241</td>
<td>5,90%</td>
</tr>
<tr>
<td>4</td>
<td>Evergreen Group</td>
<td>486908</td>
<td>156</td>
<td>5,60%</td>
</tr>
<tr>
<td>5</td>
<td>Hapag-Lloyd + CP Ships</td>
<td>424025</td>
<td>135</td>
<td>4,80%</td>
</tr>
<tr>
<td>6</td>
<td>CSCL</td>
<td>352483</td>
<td>125</td>
<td>4,00%</td>
</tr>
<tr>
<td>7</td>
<td>Hanjin</td>
<td>328794</td>
<td>84</td>
<td>3,70%</td>
</tr>
<tr>
<td>8</td>
<td>COSCO Container L.</td>
<td>322326</td>
<td>126</td>
<td>3,70%</td>
</tr>
<tr>
<td>9</td>
<td>APL</td>
<td>318927</td>
<td>100</td>
<td>3,60%</td>
</tr>
</tbody>
</table>

Table 1-1: The top nine shipping companies (February 2006).

In the environment of freight transportation it is not sufficient for shipping companies to take the control of the maritime network. Customers prefer dealing with a carrier supplying high-quality door-to-door transportation, instead of contracting several times for the different segments of the whole trip. As a consequence, shipping companies need to increase their market share and be involved in other transportation modes. Nowadays they organize the so called carrier haulage service, which includes both the maritime segment and the inland leg. Fundamentally, they sell door-to-door transportation to customers and buy land transportation from inland carriers, such as railways and trucking companies, according to rates stipulated in past contracts (Lopez, 2003).

When customers are not completely satisfied with carrier haulage services, the merchant haulage represents a practical alternative. In this case they contact railways and trucking companies separately for customized land transportation and shipping companies for the maritime leg.
The competitive environment of freight transportation forces shipping companies to achieve high performance levels in terms of management of their human and material resources. Moreover, to become engaged in intermodal transportation and provide different simultaneous services, they have to manage a huge park of containers, resulting in a significant challenge to ensure their effective control. Although managerial problems are typically thought of as decisions to move containers from one place to another, daily problems involve a wide array of choices such as renting/selling equipment, repairing, cleaning and storing containers and, wherever possible, substituting one container type for another.

The need to avoid ad-hoc decision making processes is especially crucial in the context of empty container management. This issue occurs because locations where containers are requested and delivered are normally not the same and commodities imported and exported sometimes cannot be moved in the same type of container. As a result, empty containers must be repositioned to ensure the continuity of shipping companies activity and take advantage of future transportation opportunities. Failure to provide sufficient containers exposes shipping companies to the threat of competitors who are able to provide empties as requested.

In this context it is worth noting that several optimization approaches, based on Operation Research techniques, have been recently developed to effectively support decision-making processes in terms of speed, flexibility and reliability. A large number of applications have been undertaken in the domain of transportation, resulting in substantial savings on transportation costs, that nowadays represent a relevant component of products final cost (Cordeau and Laporte, 1996; Crainic and Laporte, 1997). The successful application of Operation Research methods mainly depends on the development of more realistic models taking into account several aspects of real-world problems, the implementation of efficient algorithms and the availability of more performing computers. As a consequence, it is possible to solve large-size problems within reasonable computing effort, involving more information than in the past, enhancing the analysis of distribution problems and providing high-quality solutions. Some Operation Research methods are proposed in this thesis to support the reposition of empty containers.
1.2 Scopes, objectives and contribution

This thesis aims to provide a large overview of current logistic practices concerning the management of empty containers in the context of international trade. Moreover it investigates the reasons why their reposition represents a crucial and challenging issue for the shipping industry. Taking into account information collected through surveys and meetings with industrial experts, new optimization models are developed to support empty container management from the point of view of shipping companies.

It is worth noting that nowadays it is still not possible to adopt a general optimization model for every application in which empty containers need to be repositioned (Crainic et al., 1993-b). As a consequence, the contribution of this thesis is to model the specific features of some real-world applications properly. Compared to previous contributions in the field, such optimization models exhibits several sources of originality. First of all, the reposition among importers and exporters by truck is studied without resorting to complex Vehicle Routing formulations, that sometimes require relevant modellistic efforts and cannot be optimally solved by exact algorithms. Next, the allocation among depots is modelled considering both company-owned and rented containers through new variables and constraints. Hence this thesis aims to add realism to the maritime reposition issue, taking into account several statistically independent scenarios.

Furthermore, the thesis indicates how such optimization models cooperate, taking part in a decision support system, which coordinates empty container reposition at local and global level.

Although some optimization models were already adopted to manage the distribution of empty containers, a large part of articles in the field goes back in the sixties and seventies. Several integer programming algorithms have been investigated and evaluated to propose realistic models, that involve a huge number of variables and constraints (Abrache et al., 1999). Taking into account recent advances in the hardware technology, this thesis devotes special attention to the contribution of different resolution techniques in order to provide exact solutions within computing times suitable for the needs of the shipping industry.

Despite their possible huge size, optimization models exhibits some strong algebraic structures, sometime nested in different way, depending on the specific application. This thesis emphasizes these structural features, like networks, commodities and
scenarios, because they can be exploited to develop specialized resolution techniques, based on decomposition algorithms, lagrangian relaxations and structured interior point methods.

1.3 Outline of the thesis

Chapter 2 describes the generation of loaded and empty flows in the context of container-based distribution of goods. Due to the high standards of service imposed by the freight transportation industry, it is not possible for shipping companies to manage loaded containers only. Moreover this chapter explains the reasons why shipping companies have been forced to reposition several empty containers in recent years. Some statistics are provided to show that this activity is expected to worsen in the future. Since empty container reposition is an unavoidable activity that generates no immediate revenues, it is important for shipping companies to minimize costs associated with their management. Some Operation Research methods have been proposed in previous years. An extensive revision of the most important contributions in this field is presented.

Chapter 3 proposes a first optimization model for empty container management. Since directional imbalances in trade activities result in a surplus or shortage of empty containers in different parts of the world, their management can be thought of as a network problem whose arcs represent balancing flows, inventory links and decisions concerning the time and place to lease containers from external sources. A possible case study arising in the Mediterranean area is proposed. Solutions are also shown using a graphical representation, which provides a friendly support to the complex task of decision-makers in the field.

Chapter 4 aims to evaluate the distance between the model described in previous chapter and the current logistic practices performed by shipping companies in terms of empty container management. The purpose of this chapter is to present a detailed description of their complex activity, based on information collected through interviews with several experts of the shipping industry. Significant aspects of current logistics practices are highlighted to develop new optimization models regarding the street-turn strategy (Chapter 5), the local management of empty containers (Chapter 6) and the maritime reposition issue (Chapter 7).
The street-turn option represents a major strategy for the profitability of shipping companies supplying container-based transportation. This option is based on the allocation of empty containers by truck from importers to exporters, without first returning to an intermediate depot. In order to call for the routes to be covered by trucks, the scientific literature has proposed several methods in the context of the so-called Vehicle Routing Problem. Since sometimes this class of problems cannot be optimally solved in times suitable for the operating needs of shipping companies, chapter 5 proposes a new optimization model that exploits empty flows to determine the routes of trucks. Then decisions made by a real shipping company are compared to instances solved by Cplex (1995).

However, sometimes the street-turn strategy cannot be implemented and empty containers must be moved back and forth among customers and close depots. As a consequence, shipping companies must ensure enough containers in inland depots, where they may be requested by customers. Chapter 6 proposes two optimization models to minimize the costs of allocating empty containers among inland depots, taking into account transportation, storage and substitutions options in a deterministic and dynamic environment. While the first one considers company-owned containers only, the second one also addresses decisions on rented containers, taking into account the most significant clauses imposed by rental contacts.

When container shortage is expected in the near future and the previous options are not adequate, shipping customers can serve customers through empty containers available in ports and containers unloaded from vessels. Due to the global trade imbalance, they must perform the maritime reposition of empty containers among ports. A major difficulty in this operation is the many sources of uncertainty regarding, e.g., the number of containers that may be requested in the future, the time when empty containers become available and the vessel capacity for empty containers. Chapter 7 proposes an optimization model to solve this issue, taking into account uncertain parameters through a set of representative scenarios. Moreover the optimization model is presented in a scenario-tree formulation set up according to the “progressive hedging principle” (Rockafellar and Wets, 1991). Then a lagrangian relaxation is proposed and a bundle algorithm is adopted as non differentiable optimization technique to solve the lagrangian dual.
Conclusions are reported in chapter 8.
Chapter 2

The management of empty flows in container-based transportation

2.1 Introduction

Although several papers have addressed the planning problems of loaded containers, one of the most relevant problems for shipping companies is the management of empty containers. Unlike loaded containers, usually empty containers do not have fixed origin/destination pairs and usually no specific time schedule. This situation results in the need to plan their reposition properly in order to meet future transportation opportunities.

At a local level empty containers must be dispatched to shippers, because they need to dispatch their goods. Consequently shipping companies must guarantee enough empty containers to serve such customers in time. Moreover empties must be picked up from recipients and allocated where they may be requested. Therefore shipping companies must decide where such containers should be distributed, even if little is known about future transportation requests.

To make matters worse, directional trade imbalances result in an accumulation of unnecessary containers in import-dominant regions (the relevant case-study of the South California Region is presented) and in a shortage in export-dominant ones (for instance the Far-East). As a result, shipping companies must reposition empty containers at a global level and coordinate global and local management.

The relevance of empty containers management is generating serious problems to all operators in the shipping industry. For instance leasing companies are forced to move their empty containers where someone may be interested in this equipment. Moreover terminal containers bear high costs to provide facilities for empties.

Empty container management represents a major logistic challenge for the transportation industry, because it will never be eliminated completely. Moreover, the reduction of empty movements, besides improving the economic performance of the transportation system, will also decrease traffic problems and its environmental impact with evident benefits for the quality of life. Since shipping companies can support the allocation of empty containers by adopting advanced Operation Researcher methods, an
extended review of the most important papers in this issue is provided at the end of this chapter.

2.2 Shortcomings in the distribution of empty containers

The management of empty containers is one of the most troublesome problems for shipping companies in dealing with freight transportation in the context of international trade. In order to understand the problem properly, it is important to highlight the economic difference between loaded and empty flows. Loaded movements take place in response to customer requests, who bear transportation costs. Empty movements generate only costs and represent an unavoidable phase for the continuity of their activity.

These empty flows occur for various reasons and, due to sakes of simplicity, major attention is devoted to the container-based distribution of goods on behalf of customers. Typically a shipper, to meet the purposes of his/her industrial and commercial activities, requests shipping company to provide one or more empty containers of a given type, which must be provided to his/her location on a specific day. Then one or more trucks pick up suitable empty containers from a location close to the shipper and move such an equipment to him/her. Once containers are loaded, they are put on trucks and moved to the departure port. Usually a combination of truck and rail transportation services is requested for inland transportation over long distances. Afterward containers are loaded on appropriate vessels and moved through a series of intermediate ports to the destination port. Here they are unloaded and moved to the receiver using trucks or a combination of rail trains and trucks, according to the standard paradigm of door-to-door service. Once the final destination has been reached, containers are unloaded and moved back either to a suitable depot, where they are stored while awaiting future requests, or immediately dispatched to a new shipper.

Figure 2-1 shows the typical sequence of loaded and empty movements. Every profitable movement of a loaded container generates a non-profitable empty movement, which is however essential for the continuing operations of shipping companies. Moreover empty flows can also take place in other situations, such as in the container’s movement to/from repairing areas, to/from trade partners, to/from lessors or the purchase of new containers.
Due to global trade imbalance, shipping companies tend to accumulate empty containers in import-dominant regions where they are not needed, whereas export-dominants face a shortage of this equipment. Thus, the difference between outbound and inbound loaded containers in each region results in the need to reposition containers or find additional loads, if it is possible. In a perfect world empty movements would not exist, because there would always be cargo to fill every container when and where it is emptied.

However, commercial traffic never seems to be in balance, either in volume or value, and shipping companies must reposition empty containers on global, national and local scales.

Although trade imbalances have always existed, some reasons have forced the need of reposition a relevant number of empty containers in past years. In 1998 economic pressures have reduced the buying power of many Asian currencies, making Asian goods cheaper for American and European consumers and western ones more expensive in the Far East. As a consequence, there has been a reduction in Asian imports, an increase in Asian exports and an impressive number of empty containers have been stranded in Europe and the west coast of the U.S.A..

Such reasons are clearly stated in Figure 2-2 which shows in the upper graph the monthly difference between discharged and loaded containers on the West Coast of USA, while the lower one indicates the total number of containers.
It is worth noting that during 1995 and early 1996 inbound and outbound loaded containers were nearly in balance, and for several months loaded TEUs exceeded discharged TEUs. This period corresponded closely to a time when the Japanese yen and other Asian currencies were very strong against the U.S.A dollar.

Due to the steady growth of exports from China, directional imbalances in trade are expected to continue and even worsen, taking quite some time. Therefore this trend results in a strong mismatch between North-American and European ports, where empty containers tend to be accrued, and those of the Far East, where this equipment needs to be reallocated.

In order to effectively represent directional imbalances in container-based commerce, the movements of loaded flows along three main East-West trades are presented, according to information provided by Mitsui O.S.K. Lines (http://www.mol.co.jp). The difference among westbound and eastbound flows represents the number of empty containers to be repositioned.

Table 2-1 refers to the quarterly data regarding trade between Asia and North America. Typically in the past few years, an average of 40% to 50% of loaded containers shipped from far-East to the West Coast of the U.S. were in the end moved back as empty containers. In May 2006 the eastbound cargo volume (EB) exceeded for the first time the million of TEUs, amounting to 1108559 TEU. In the other direction, the westbound volume (WB) was 390397 TEU and the imbalance ratio WB/EB attained the 35,2%.
Table 2-1. Directional imbalances in Asia/North America trade.

Table 2-2 concerns quarterly data on trade between Asia and Europe. Although in past years Europe has always exported more containers than it imports, nowadays westbound traffics significantly overcome the eastbound ones.

Table 2-2. Directional imbalances in Asia/Europe trade.
However, this table does not clearly state that goods transferred in the two directions often need a different container type. Indeed, 20-foot containers are mostly used in eastbound freight flows, while the westbound traffic usually calls for 40-foot containers. As a consequence, a suitable mathematical model for container transportation should take into account different container types and, wherever it is possible, address the opportunity of performing substitutions.

Table 2-3 refers to the trade between Europe and North America. In recent years trade imbalance has been partially alleviated by the appreciation of the Euro against the USA dollar. As a consequence, European goods have become more expensive in the USA and US goods cheaper for European consumers.

![Graph showing container traffic between North America and Europe](image)

Table 2-3. Directional imbalances in North America - Europe trade.

Transporting air in a heavy steel box from one part of the world to another is surely not a profitable way to do business because it involves important logistic costs for carriers, resulting in significant impacts for the performance of transportation system. In 1999, it was estimated that the cost of managing empty containers amounted to more than US$ 25 billion worldwide and that, if that tendency were to persist, this cost would exceed US$ 50 billion a year by 2010. Moreover, the cost of providing new facilities to store and handle empty containers would call for billions more (Olivo et al., 2003).

Since the resources spent for such laborious activity have a primary importance for container-based trade, it is not surprising that several shipping companies are examining
the possibility of reducing these unproductive costs, which are essential for the continuity of their activity. In a time of slim profit margins, proper management of empty containers (within the limits imposed by demand and service requirements) may well represent a key factor for the competitiveness of carriers or, at least, may allow their survival in the market. Moreover, since empty flows are simply a loss, shipping companies cannot afford to bear such expenses without shippers expecting higher rates for loaded movements.

Empty containers are also creating relevant problems to American and European terminals, that have been forced to organize their storage in high blocks. However, the accumulation of empties, which have longer dwelling times than loaded ones, is not an asset for ports, especially when land availability is scarce. Sometimes they have to create new facilities in order to ensure their storage and, as shown by the occurrences in Singapore, Hong Kong and Rotterdam, the expansion of terminal containers costs many millions of dollars, involving the occupation high-value lands, adjacent to ports, which otherwise could be used more profitability.

Empty containers have caused a significant trouble for lessors and leasing companies as well, which possess more or less 50% of the world container fleet. The reasons of their difficulty depends both on the directional imbalances of trade traffic and specific problems in their industry. The delivery of larger vessels and stagnation in trade has provided shipping companies an edge over them, because carriers can use the available slots of their vessels to reposition their own empty containers. Moreover, in 2000 and 2001, shipping companies ordered new containers, thus reducing the need to lease new ones. What is more, shipping companies have a surplus of their own containers and, as a consequence, have diminished the demand for on-hire services, which results in a massive returning of containers to their owners.

In this situation, many leasing companies have significantly invested to reposition empty containers from high cost depots, situated in European and American regions, to less onerous storage areas of China and other regions of Far-East. Such regions offer by far more favorable opportunities for their containers. Although transportation from North America to Asia can cost from 500 to 800 U.S.A. dollars per TEU, many tens of thousands of empties were repositioned by leasing companies during 2000, 2001 and 2002. Nowadays, the largest leasing companies currently spend something like 20-30
million U.S.A. dollars a year for the chartering of vessels to reposition their containers (http://www.informare.it).

Such idle equipment represents a serious burden because it does not only produce any return, but it also involves unavoidable and exorbitant warehouse costs, resulting in a negative flow of revenues. These costs vary depending on the location of depots and, in the worse cases, they can totally cancel the daily proceeds deriving from the leasing of other containers. Moreover lessors and leasing companies press shipping companies to return empty containers in export dominant regions, where new business opportunities can be achieved. Such clauses are often included in contracts, even when they last for several years, forcing shipping companies to reposition leased containers as well.

Empty container distribution poses some problems on the regional perspectives as well. Hanh (2003), focusing on the specific situation of Southern California Region, argues that current logistic practices seem rather inefficient and illogical, because, once having reached final destination, empty containers are moved to ports and, when new transportation opportunities arise, they are hauled back to the shipper. The reasons why empty containers move back and forth in such a competitive industry depends on the greater relevance of global logistic over regional solutions. Drawbacks related to equipment shortage for Asian shippers prevail by far over the possible benefits of an efficient distribution of empty containers in the region. Costs saved to reposition empties in the Far-East do not encourage shipping companies to risk the reuse of containers for local export.

In his opinion, for shipping companies the reuse of empty containers is an attractive solution to rationalize the movement of empty containers only if they are finally moved to a an export-dominant area. A major barrier to containers reuse is represented by the so-called Interchange (or Equipment Interchange Receipt), that is the transfer of equipment from the responsibility of an operator to another, like railways, trucking companies and intermediates (Lopez, 2003). During interchanges containers are inspected to establish responsibility for any damage and, since shipping companies do not know before the maintenance status, this check introduces uncertainty on equipment availability and decreases the control of shipping companies over their containers. This problem becomes even more relevant for containers transiting among multiple users. This partial lack of visibility certainly means inefficient exploitation of resources and
forces shipping companies to perform low-risk maritime reposition of containers towards the Far-East, because maritime transportation takes place over their care.

Potential support to container reused can be provided by promising web-based information and communication systems. To improve the quality of information and transform such communication systems into practicable solutions supporting fleet management, shipping companies need not only a simple distribution of information, but correctness and reactivity of data as well. As a consequence, effective usage of such systems needs the cooperation of all operators involved in intermodal freight transportation, who are expected to share information.

Fundamentally, the freight transportation market needs to promote cooperation between the several links of the logistic chain. Their trade alliances will allow transportation firms to improve customer satisfaction and avoid empty container flows. The sharing among several companies of containers without logos, the so-called grey containers, is a development of this logic. Moreover an improvement of loading factor can be achieved, since nowadays each container is assigned to a single customer.

Finally, a forthcoming strategy to save transport, transshipment and storage costs may be the possibility to fold containers. Konings (2005) examined the commercial opportunities of foldable containers and showed they can lead to important benefit in the transportation chain. However, on the basis of a cost-benefit analysis, he pointed out that much depends on the additional costs, such as handling, getting specialized equipment and moving containers to places where folding and unfolding can take place. Foldable containers are not expected to be assembled or disassembled easily anytime and anywhere. Substantial benefits would arise when economies of scale can be achieved (for example during long-haul transportation). Moreover, the number of locations where folding/unfolding would take place should be limited to build up specialized expertise reducing the costs involved.

Even if foldable containers look promising, they are not extensively adopted in practice. Past experiences showed higher purchase prices, vulnerability to damages and frequent maintenance costs. Moreover the process of mounting and dismounting is time-consuming, especially when some damages occur, making their use prohibitive. Last but not least, their high weight represents a major barrier to market introduction, because it makes the movement of folded containers a difficult task.
Major attention in the thesis is therefore devoted to methods aiming to minimize reposition movements for standard containers that cannot be folded.

2.3 A review of optimization models for the management of empty flows

The movement of empty containers generates no profits for shipping companies, but it represents an essential operation to satisfy future transportation opportunities. As a result, shipping companies seek to minimize costs associated with empty container management, within the general restriction imposed by service requirements. This paragraph recalls, chronologically, several operational models in this issue, emphasizing the evolution of their assumptions.

A significant part of these contributions falls into a similar issue, the empty railroad car allocation problem, as models developed for rail have been often extended to other transportation modes and more complex multimodal networks. Early linear models minimized the total cost of moving a homogeneous fleet of empty cars (Leddon and Wrathall, 1967; Misra, 1972; Baker, 1977). The simplex algorithm was adopted as a solution method.

Previous contributions presented static optimization models and took into account estimation of uncertain parameters. To propose more realistic formulations, an important contribution was the introduction of the time perspective. Early efforts in this direction resulted in deterministic dynamic network optimization models, to manage homogeneous fleets of empty cars (White, 1968; White and Bomberault, 1969; Ouimet, 1972). These papers proposed an out-of-kilter algorithm as a resolution technique (Fulkerson, 1961).

Herren (1973, 1977) represented the empty car distribution issue as a minimum cost model on a dynamic network. Several car types were taken into account. Each node represented a car on a train and arcs characterized switching operations. He solved the model by adapting an out-of-kilter algorithm providing a substantial drop in transportation costs.

The explicit consideration of uncertainty in demand and supply was introduced by Jordan and Turnquist (1983). They proposed a multicommodity dynamic model, in which supply, demand and travel times represent stochastic parameters. Taking into account known and forecasted parameters, their formulation determined the decisions
for the current time and anticipated decisions of future periods. As time advances, new information arrives and the model is solved again using updated data. Their formulation took the form of a nonlinear optimization model with linear constraints and it is solved by an effective Frank-Wolfe algorithm (1956). Afterwards more complex formulations were introduced to account for more realistic conditions, like substitution options, integer flows and economies of scale (Shan, 1985, Chih, 1986, Markowicz and Turnquist, 1990; Turquist, 1994). Additional restrictions on empty vehicles were imposed to model specific case studies. For example, Holmberg and al. (1998) examined the case of Swedish State Railways. They proposed a dynamic multicommodity network flow model. By taking into account some transportation capacity constraints for empty containers, this model can reduce the shortcomings in their distribution, such as high repositioning costs and low utilization of freight cars. Joborn et al. (2004) proposed a dynamic optimization model that explicitly took into account economies of scale in the distribution of empty cars. The modelling of these economies of scale was performed by fixed costs associated with some space-time paths, in addition to costs proportional to flows. They proposed an efficient tabu-search algorithm, that significantly outperformed the branch-an-bound algorithm provided by a commercial software.

Moving the focus onto empty containers, the first formulation can be found in Potts (1970). White (1972) proposed a deterministic dynamic transshipment network, solved by the out-of-kilter algorithm. Ermol’ev et al. (1976) presented a time-extended network model for the maritime reposition of empty containers. A network optimization algorithm was proposed for this purpose. Florez (1986) built a deterministic dynamic profit-maximizing model for the empty container maritime repositioning and leasing. Several instances were solved by two linear programming algorithms. According to Dejax and Crainic (1987) the management of empty flows has received much attention since the sixties. However few contributions have been proposed to develop specific models addressing the empty container allocation issue. Crainic and al. (1989) described the strategic issue of assigning empty containers to depots. They proposed a multimode multicommodity location formulation with inter-depot balancing requirements, to minimize the cost of depot opening and empty container transportation. Several solution procedures were proposed: sequential branch-and-bound (Crainic et al.,

Crainic and al. (1993-b) proposed a framework for the empty container allocation problem in the context of an inland distribution system managed by an international maritime shipping company. They addressed the specific characteristics of this issue and developed deterministic dynamic models in single commodity and multicommodity cases aiming to minimize total inland operating costs. Moreover, they provided a mathematical formulation to deal with the uncertain feature of demands and supplies. Abrance et al. (1999) proposed a decomposition technique for the multicommodity version of previous paper, that exploited the underlying network structure. Several strategies were derived from it and implemented in sequential and parallel environments. Lai et al. (1995) represented the dynamic and stochastic container management problem using simulation. They evaluated several allocation policies to prevent the shortage of empty containers when they are requested. However, the huge size of problems resulted in a problematic testing phase. Shen and Khoong (1995), focusing on the business perspective of the shipping industry, developed a deterministic dynamic network optimization model for empty container management. It is able to minimize repositioning costs and provide decisions about leasing and containers returning from external sources. Since, in the real-world, it is necessary to deal with the stochastic systems, Cheung and Chen (1998) applied the recourse formulation to the dynamic empty container allocation problem in the context of an international maritime system. Uncertainty derived from transportation capacity for empties, demands for and supplies of empty containers. Moreover, they conducted numerical tests to assess the value of the stochastic model over a rolling horizon environment. Choong and al. (2002) addressed the planning horizon length issue for intermodal transportation networks. They developed a deterministic integer programming model and concluded that the application of a longer planning horizon usually allows a better distribution of empty containers, encouraging the use of slow and cheap modes.
Leung and Wu (2004) developed a robust optimization model for the dynamic reposition of empty containers from surplus ports to demand ports with uncertain customer demands. Compared to stochastic programming, their contribution resulted in solutions slightly insensitive to the different realizations of uncertain parameters. A specific weight sets up the trade-off between expected costs and risk.

Erera et al. (2005) proposed a deterministic time-extended large-scale optimization model to simultaneously manage loaded and empty tank containers. The incorporation of routing and reposition decisions in a single multicommodity model resulted in a significant lessening of operating costs. Several computational tests were solved in times suitable for the needs of the tank container operator.

The simultaneous management of loaded and empty vehicles/containers to maximize total profits has been extensively investigated to address decisions of trucking companies. However, this thesis focuses on the point of view of shipping companies, that tend to outsource trucking transportation. Anyway many details are available in the papers by Powell (1988, 1996).

However, to maximize their profits, shipping companies are forced to minimize empty trips for their trucking transportation providers. Since trucking companies must be instructed about routes to be covered, the specific issue faced by shipping companies takes the form of the so-called Vehicle Routing Problem with Backhauls (Toth and Vigo, 2002). Further details are provided in Chapter 5.
Chapter 3

An operational model for empty container management over intermodal networks

3.1 Introduction

The aim of this chapter is to propose an optimization model for shipping companies in dealing with the operational management of empty containers. Moreover, this study presents effective algorithms adopted to optimally solve several instances. According to the container distribution issue described in Chapter 2, main attention is devoted to answer these questions properly:

- How should shipping companies provide empty containers to shippers?
- Where should shipping companies move containers returned by recipients?
- When should the shipping companies on-hire empty containers?
- When should the shipping companies off-hire empty containers?

Literature in this field has proposed several dynamic optimization models, but few contributions were developed in the context of real-time management of heterogeneous fleets of empty containers (Crainic, 2003). Furthermore modern technology allows transmitting real-time information regarding the state of cargo and adopting an operating platform common to different intermodal partners. As a consequence, operational management can be properly represented by using an hour rather than a day as time-step of a dynamic model.

The reasons why such a short temporal unit has not been adopted before may depend on the opinion that it might lead to computationally intractable models. However, taking into account the available computational speed and the recent diffusion of new algorithms, this study aims to adopt such a period of time as time-step in a weekly planning horizon.

Moreover, hourly resolution provides an adequate representation of the transportation systems evolution over time and enables decision-makers to promptly correct the system as soon as unexpected information arrives (e.g. accidents and equipment breakdowns). Moreover, hourly time-steps allow selecting which available transportation services should be used to reposition empty containers on a given day.
Since transportation services permit the reposition of a limited number of empty containers, all variables of the proposed dynamic model are upper bounded. Although such a situation may cause some infeasible situations, this contribution describes a method that always returns feasible solutions. Such a method was not yet detailed in previous literature in the field.

Dejax and Crainic (1987) noted that, although the allocation of empty vehicles has received much interest since the sixties, little attention has been dedicated to the development of innovative models addressing specifically the container transportation issue. They mentioned a few authors who, focusing only on the maritime operations of shipping companies, had addressed the problem of allocating empty containers available at a surplus terminal to a demanding terminal, in preparation for subsequent transport of loaded containers, using linear programming formulations in a deterministic and dynamic approach.

Crainic et al. (1989) presented the problem of locating empty vehicles in an intercity freight transportation system in order to minimize the cost of depot opening and vehicle transportation, while satisfying customer requests. For this purpose, they developed a two-echelon multimode multicommodity location-distribution model with inter-depot balancing requirements. Such an approach is suitable for a strategic planning level because it optimizes long-term decisions such as terminals to be used in repositioning empty containers. The space-time dependency of events was not considered.

Crainic et al. (1993-b) presented a general framework for the empty container allocation problem in the context of a land distribution system for an international maritime shipping company, addressing specific characteristics of this issue. In order to obtain computationally tractable models, taking into account the large number of decision variables in any period, they proposed the day as time-step and suggested limiting the length of the planning horizon to 10-20 days. No computational result was provided.

Shen and Khoong (1995), focusing on the business perspective of the shipping industry, developed a network optimization model for empty containers to minimize repositioning costs and provide decisions covering leasing and returning containers from external sources. They used the day as time-period of a dynamic network executed in a rolling horizon fashion, but no experimental result is presented.
Since in the real world decision-makers must deal with the stochastic systems, Cheung
and Chen (1998) applied the recourse formulation to the dynamic empty container
allocation problem in the context of international maritime system. They compared the
value of a two-stage deterministic model with a two-stage stochastic model. They
considered a single-commodity dynamic network and assumed that one time-period
represents one day. In the first part of the planning horizon they used a deterministic
network of 7 days, and in the second a deterministic or stochastic network with variable
length. Some numerical tests were performed to assess the value of the stochastic model
over a rolling horizon environment.

Distribution planning of empty vehicles occurs in other transportation areas, such as
empty freight car distribution in a railway transportation system described by Holmberg
et al. (1998). To reduce the shortcomings of this planning process, they proposed a
dynamic multicommodity network, assuming that all car types have the same length.
The planning horizon is divided into a set of time-periods, in which each period can be a
day or part of a day, and a rolling horizon of 10 days is implemented. They also
suggested that the planning horizon should be longer than the longest transportation
time in the network.

Jiele (1999) investigated the empty container distribution problem for single commodity
cases with minimum cost flow algorithm and multicommodity variance using a linear
programming technique. The author considered two container types of the same size in
the multi-commodity model, thus both types can fill the same empty slots. He proposed
the day as time-period and, in order to illustrate the relation between problem size and
run time, he considered planning horizons varying between 5 and 1000 days.

Choong et al. (2002) addressed the end-of-horizon issue in the management of empty
containers for intermodal transportation networks. They adopted the day as time-period
and evaluated model sensitivity to the length of planning horizon comparing a 15-day
model to a 30-day model. In order to handle infeasibilities, they changed supply and
demand of “unreachable” locations to zero, while this contribution shows that dummy
nodes and arcs offer a better interpretation of results.

The outline of this chapter is as follows. In Section 3.2 an optimization model for
managing empty containers over a weekly planning horizon is presented. Section 3.3
describes modellistic methods used to avoid unfeasible solutions. Section 3.4 presents a
case study. Several instances are solved by adopting both commercial and freeware algorithms.

3.2 Optimization model

The proposed mathematical model seeks to minimize the overall cost of managing empty containers on a continental scale. This study considers a managerial environment made up of several regional shunting points sending information to a central authority. This information regards the number of empty containers supplied and demanded in each region and time-period of the planning horizon. In order not to lose the temporal data resolution, when we mention a given number of containers required in a region in a given time-period, we do not mean that customers want to receive that number of containers in that period of time, but rather that empty containers must reach this shunting point in such a period in order to satisfy customers requests promptly. In the same way, when we mention a given number of containers supplied in a given time-period in a region, we do not mean that customers release that number of containers in that time-period, but rather that this number of containers is declared available in such a shunting point in this period of time.

In order to derive the local demand with hourly resolution, we need to know the customer requests and take into account the transit time (i.e. transportation time plus a given component depending on the activity of dispatching points) within the local area to reach the customer’s location. Since customer requests are typically known at the beginning of the weekly planning horizon, we can convert such information into hourly orders.

Since carriers are not able to know at the beginning of the planning horizon when every container will become available, they must forecast the local supply of empty containers in each hour of the planning horizon. Although this is not a straightforward process, they can predict the most probable number of containers that will be ready for orders in each time-period of the planning horizon using their statistical basis.

In order to face uncertainty, this study adopts a rolling horizon procedure. To clarify, we solve instances using as information the number of containers available at each shunting point at the beginning of the planning horizon and a significant sample of containers that will become available over time. Then we implement decisions for the first time-
period and, when in the following hourly interval we receive new information about containers available by the end of the planning horizon, we adjust data and reoptimize the system and so on.

The rolling horizon procedure is also used to deal with further sources of uncertainty such as transit time and equipment failures. Since transit time can be modified by traffic jams and terminal delays, we first solve the problem using the expected transit times and, when new information about delays arrives in the following hourly interval, we correct data and reoptimize. Finally, equipment failures and accidents (that may have a serious impact on the performance of a transportation company) are managed in the same way. It is worth noting that, since it is essential for carriers to adjust quickly previous decisions when such events occur, a hourly resolution is more appropriate than a daily one.

Moving the focus onto the notation used in this study, we consider three transportation modes (lorry, rail and ship), a set $Q$ composed of two commodities (20 ft and 40 ft containers) and a set $T$ of contiguous time-periods. As described above, the area of study is shared in zones, each managed by an authority and characterized by a “macronode”, representing a shunting point where regional supply and demand for empty containers is aggregated and compensated. To clarify, regional surpluses and deficits are calculated as the difference between the local supply and demand for empty containers. Then the demand for empty containers in a certain region, in a given period of time for a given container type is - at least partially - satisfied by the number of empty containers of that type supplied in the same zone in that period of time.

The proposed model is based on a graph $G=(D \cup N, A)$, consisting of a finite set of nodes $D \cup N$ and a set $A$ of directed arcs. Each node represents a depot $D$ or macronode $N$ for each commodity $q \in Q$ and each time period $t \in T$. Each arc $(i, j_{t+\tau})$ represents a link between $i \in D \cup N$ in time-period $t \in T$ and $j \in D \cup N$ in time-period $t+\tau \in T$ ($\tau$ is the transit time between $i$ and $j$). Every arc $(i, i_{t+1})$ denotes the inventory of containers in depot $i \in D$ between time-periods $t \in T$ and $t+1 \in T$. The notation $b_{(i, q, t)}$ for a macronode $i \in N$ represents the supply of empty containers of commodity $q \in Q$ in time-period $t \in T$, if $b_{(i, q, t)}$ is positive, or the request, if $b_{(i, q, t)}$ is negative. To clarify, if the local demand for a given commodity $q \in Q$ and a given time-period $t \in T$ is greater than the
corresponding supply, in macronode \( i \in N \) there is a deficit of empty containers and the related \( b_{i(q,t)} \) is negative. If the local supply, given \( q \in Q \) and \( t \in T \), is greater than the local demand, there is a surplus in that macronode and the related \( b_{i(q,t)} \) is positive.

Depots, excluding the first and last time-periods, are trans-shipment nodes \( (b_{i(q,t)} = 0, i \in D, q \in Q \) and \( t \in T \)\), indeed they can receive empty containers from supply macronodes and other depots, allocate empty containers to demand macronodes and other depots, keep the previous period stock and store empty containers for the subsequent time-period. We also suppose that the number of empty containers supplied in a depot at the beginning of the current planning horizon is equal to the number of containers required in the same depot in the last period of planning horizon. The following assumptions are also made:

- For each commodity \( q \in Q \) and each time-period \( t \in T \) a significant sample of the number of empty containers supplied in each macronode is known.
- For each commodity \( q \in Q \) and each time-period \( t \in T \) the number of empty containers required in each macronode is known.
- Empty containers cannot be held at the supply macronode. After unloading, empty containers are moved to demand macronodes or depots.
- Delivery windows are not included: containers are delivered to the demand macronode in the time-period in which they are needed.
- Empty containers can be stored only in depots.
- No backorders and substitution of commodities are allowed.
- Leased containers are modelled as company-owned containers.
- All costs are independent of time-periods. This assumption can be relaxed without causing problems.
- Storage capacity of container depots is dependent on time-periods.
- The capacity of railway and maritime links depends on time-periods, according to available transportation services.
- Containers are ready for use and no repairs or discards occur.

This model also takes into account the capacity of storage pools and transportation modes. A sample network is proposed in Figure 3-1, where macronodes are indicated with numbers from 1 to 7, depots with 8 and 9 and some arcs link the nodes.
However, the explicit representation of the multi-period perspective is provided by Figure 3-2, where five time-periods are considered and the following notation is used:

- $S_{i(q,t)}$: supply in node $i \in D \cup N$ of empty containers of commodity $q \in Q$ at time $t \in T$ ($b_{i(q,t)} > 0$).
- $R_{i(q,t)}$: request in node $i \in D \cup N$ of empty containers of commodity $q \in Q$ at time $t \in T$ ($b_{i(q,t)} < 0$).

The problem is represented as an integer programming model, whose decision variables are the flows of empty containers between the nodes of the space-time network. Eight main types of decision variables are considered, denoting transportation modes, the departure and arrival times. We denote with letter $X$ the 20 ft commodity, with letter $Y$ the 40 ft, with $C$ costs, with $U$ upper bounds, with $L$ the lorry transportation mode, with $R$ railway mode, with $M$ the maritime mode and with $H$ the holding of containers in depot.

Hereafter, the list of such decision variables, costs and upper bounds is presented:

1. Vector $XL(i, j, \tau)$ denotes the flow of 20 ft empty containers using lorry transportation mode from node $i \in D \cup N$ in time-period $t \in T$ to node $j \in D \cup N$ in time-period $t+\tau \in T$ ($\tau$ is the transit time between $i$ and $j$), $CXL(i, j, \tau)$ the relative cost per unit of flow and, assuming that one 20ft or 40 ft container is hauled by each lorry, $UL(i, j, \tau)$ denotes its upper bound expressed in number of lorries.
Figure 3-2. The multi-period representation of the simple network.
2. Vector $XR(i,j_{t+\tau})$ denotes the flow of 20 ft empty containers using rail transportation mode from node $i \in D \cup N$ in time-period $t \in T$ to node $j \in D \cup N$ in time-period $t+\tau \in T$ ($\tau$ is the transit time between $i$ and $j$), $CXR(i,j_{t+\tau})$ the relative cost per unit of flow and $UXR(i,j_{t+\tau})$ its upper bound expressed in number of 20 ft containers.

3. Vector $XM(i,j_{t+\tau})$ denotes the flow of 20 ft empty containers using maritime transportation mode from node $i \in D \cup N$ in time-period $t \in T$ to node $j \in D \cup N$ in period $t+\tau \in T$ ($\tau$ is the transit time between $i$ and $j$), $CXM(i,j_{t+\tau})$ the relative cost per unit of flow and $UXM(i,j_{t+\tau})$ its upper bound expressed in number of 20 ft containers.

4. Vector $XH(i,i_{t+1})$ denotes the inventory of 20 ft empty containers stored in depot $i \in D$ between time-period $t \in T$ and $t+1 \in T$, $CXH(i,i_{t+1})$ the relative cost per unit of flow and $UXH(i,i_{t+1})$ its upper bound expressed in number of 20 ft containers.

5. Vector $YL(i,j_{t+\tau})$ denotes the flow of 40 ft empty containers using lorry transportation mode from node $i \in D \cup N$ in time-period $t \in T$ to node $j \in D \cup N$ in time-period $t+\tau \in T$ ($\tau$ is the transit time between $i$ and $j$), $CYL(i,j_{t+\tau})$ the relative cost per unit of flow and $UL(i,j_{t+\tau})$ its upper bound expressed in number of lorries.

6. Vector $XR(i,j_{t+\tau})$ denotes the flow of 40 ft empty containers using rail transportation mode from node $i \in D \cup N$ in time-period $t \in T$ to node $j \in D \cup N$ in time-period $t+\tau \in T$ ($\tau$ is the transit time between $i$ and $j$), $CYR(i,j_{t+\tau})$ the relative cost per unit of flow and $UXR(i,j_{t+\tau})$ its upper bound expressed in number of 20 ft containers.

7. Vector $YM(i,j_{t+\tau})$ denotes the flow of 40 ft empty containers using maritime transportation mode from node $i \in D \cup N$ in time-period $t \in T$ to node $j \in D \cup N$ in period $t+\tau \in T$ ($\tau$ is the transit time between $i$ and $j$), $CYM(i,j_{t+\tau})$ the relative
cost per unit of flow and $UXM(i_i,j_{t+\tau})$ its upper bound expressed in number of 20 ft containers.

8. Vector $YH(i_i,j_{t+\tau})$ denotes the inventory of 40 ft empty containers stored in depot $i \in D$ between time-period $t \in T$ and $t+1 \in T$, $CYH(i_i,j_{t+\tau})$ the relative cost per unit of flow and $UXH(i_i,j_{t+\tau})$ its upper bound expressed in number of 20 ft containers.

Being $F(i,q,t):\{j : (i,j) \in A, q \in Q, t \in T\}$ and $B(i,q,t):\{j : (j,i) \in A, q \in Q, t \in T\}$, the optimization model may be expressed as follows:

$$
\min \sum_{t \in T} \left\{ \sum_{i \in D} \sum_{j \in D} (CXL(i_i,j_{t+\tau}) \cdot XL(i_i,j_{t+\tau}) + CXR(i_i,j_{t+\tau}) \cdot XR(i_i,j_{t+\tau}) + CXM(i_i,j_{t+\tau}) \cdot XM(i_i,j_{t+\tau}) + CYL(i_i,j_{t+\tau}) \cdot YL(i_i,j_{t+\tau}) + CYR(i_i,j_{t+\tau}) \cdot YR(i_i,j_{t+\tau}) + CYM(i_i,j_{t+\tau}) \cdot YM(i_i,j_{t+\tau}) + \sum_{i \in D} (CXH(i_i,i_{t+\tau}) \cdot XH(i_i,i_{t+\tau}) + CYH(i_i,i_{t+\tau}) \cdot YH(i_i,i_{t+\tau})) \right\}$$

(3.1)

$$
\sum_{j \in F(i,q,t)} \left\{ XL(i_i,j_{t+\tau}) + XR(i_i,j_{t+\tau}) \cdot XM(i_i,j_{t+\tau}) \right\} + XH(i_i,i_{t+\tau}) - XH(i_{t-1},i_{t}) = b_{i(x,t)}
$$

(3.2)

$$
\sum_{j \in F(i,q,t)} \left\{ YL(i_i,j_{t+\tau}) + YR(i_i,j_{t+\tau}) \cdot YM(i_i,j_{t+\tau}) \right\} + YH(i_i,i_{t+\tau}) - YH(i_{t-1},i_{t}) = b_{i(y,t)}
$$

(3.3)

$$
XL(i_i,j_{t+\tau}) + YL(i_i,j_{t+\tau}) \leq UL(i_i,j_{t+\tau}) \quad \forall i, j \in D \cup N, \forall t \in T
$$

(3.4)

$$
XR(i_i,j_{t+\tau}) + 2YR(i_i,j_{t+\tau}) \leq UXR(i_i,j_{t+\tau}) \quad \forall i, j \in D \cup N, \forall t \in T
$$

(3.5)
New Optimization Models for Empty Container Management

\[ X_M(i, j_{t+\tau}) + 2Y_M(i, j_{t+\tau}) \leq U_XM(i, j_{t+\tau}) \quad \forall i, j \in D \cup N, \forall t \in T \]  

(3.6)

\[ X_H(i, i_{t+\tau}) + 2Y_H(i, i_{t+\tau}) \leq U_XH(i, i_{t+\tau}) \quad \forall i \in D, \forall t \in T \]  

(3.7)

\[ X_L(i, j_{t+\tau}), \ X_R(i, j_{t+\tau}), \ X_M(i, j_{t+\tau}), \ X_H(i, i_{t+\tau}), \ Y_L(i, j_{t+\tau}), \ Y_R(i, j_{t+\tau}), \]  

\[ Y_M(i, j_{t+\tau}), \ Y_H(i, i_{t+\tau}) \geq 0, \text{ integer} \quad \forall i, j \in D \cup N, \forall t \in T \]  

(3.8)

where \( t-\tau, t+\tau, t-1 \) and \( t+1 \) must belong to \( T \).

The objective function (3.1) represents the total cost of empty container management, including cost of transportation and storage in depots. Constraint (3.2) represents the mass balance conservation of 20 ft containers at the macronode or depot \( i \) during time-period \( t \). Constraint (3.3) represents the flow conservation of 40 ft containers at macronode or depot \( i \) during time-period \( t \). In these equations \( b_{i(t,t)} \) and \( b_{i(t,t)} \) represent the supply of 20 and 40 ft containers in node \( i \) during time-period \( t \) if they are positive, or the demand if they are negative. The demand of macronodes in time-period \( t \) must be met by surplus macronodes or by depot inventories, using existing transportation modes. On the other hand, the supply of surplus macronodes must be moved toward deficit macronodes or container depots.

Constraint set (3.4) states that the total number of empty containers, hauled by lorry from node \( i \) in time-period \( t \) to node \( j \) in time-period \( t+\tau \), cannot exceed a transportation capacity, expressed with the number of lorries \( U_L(i, j_{t+\tau}) \). Since considering lorry numbers instead of container numbers may allow more accurate assessment of empty flow incidence on the roads, we assume for the sake of simplicity that each truck can transport only one container (one 20 ft or one 40 ft container). Furthermore, we assume that a truck with a 20 ft container and a truck with a 40 ft container use the same amount of capacity supplied by roads, so we use the same coefficient 1 for 20 ft and 40 ft containers.

Constraint set (3.5) states that the total number of empty containers, carried by rail from node \( i \) in time-period \( t \) to node \( j \) in time-period \( t+\tau \), cannot exceed a transportation capacity expressed with the 20 ft container number \( U_XR(i, j_{t+\tau}) \). Since a 20 ft container is half the size of a 40 ft container, the number of 20 ft containers transportable starting in time-period \( t \) is double that of 40 ft containers.
Constraint set (3.6) indicates that the total number of empty containers, shipped from node $i$ in time-period $t$ to node $j$ in time-period $t+\tau$, cannot exceed a transportation capacity expressed by the number of 20 ft containers $UXM(i,j,t,\tau)$. Since a 40 ft container fills two 20 ft container slots, the number of 20 ft containers transportable starting in time-period $t$ is double that of the 40 ft containers.

Constraint set (3.7) ensures that the stock of empty containers in depot $i$ during time-period $t$ does not exceed its storage capacity, expressed in the 20 ft container number $UXH(i,t)$. Since a 40 ft container fills two 20 ft container slots, the number of 20 ft containers that can be stored between time-period $t$ and $t+1$ is double that of 40 ft containers.

Constraint set (3.8) indicates that all decision variables can only have non-negative integer values, to ensure that fractions of containers will not be considered in the solution.

It is possible to note that the proposed model can be converted to a multicommodity problem that can be solved by several specific and efficient algorithms, by making the following substitutions:

\begin{align}
2YR(i,j,\tau) & = ZR(i,j,\tau) & \forall i, j \in D \cup N, \forall t \in T \\
2YM(i,j,\tau) & = ZM(i,j,\tau) & \forall i, j \in D \cup N, \forall t \in T \\
2YH(i,\tau) & = ZH(i,\tau) & \forall i \in D, \forall t \in T
\end{align}

\(3.9\) \(3.10\) \(3.11\)

### 3.3 A revision of the model against infeasible solutions

The previous model cannot be implemented as described, because infeasible solutions may be obtained.

First, it is necessary to meet all demands arising in the network, indeed the supply of empty containers may be lower than the total demand and arcs may not reach demand macronodes in time (for instance macronode 3 of time-period 1). To meet all deficits, it is necessary to propose a method taking into account the upper bounds associated with arcs. Depots do not have unlimited inventory capacities and no transportation infrastructure can guarantee the transit of an unlimited number of containers in the time-step. Therefore, the deficit of empty containers in every macronode must not exceed the sum of the upper bounds of arcs entering that macronode.
In order to solve this problem, we introduce a dummy node supplying or requiring a number of containers equal to the algebraic sum, changed as to sign, of all empties offered and demanded. Surpluses are denoted by the positive sign, whereas deficits by the negative. The next step is connecting the dummy node to demand macronodes using artificial arcs. As a result, the number of empty containers demanded and supplied over the network is equal and all deficits can be met. Moreover, in order to eliminate every risk of infeasibility, such arcs are unbounded above.

The use of these dummy arcs in the solution can be interpreted as an opportunity for meeting demand in macronodes using equipment borrowed, leased or purchased. Sometimes it may also be read as the payment of a fine for a failed arrival of empty containers at that macronode. Based on the different situations, it is possible to decide the most suitable costs of artificial arcs.

Supply macronodes may also cause infeasibility problems as well. It is necessary to ensure that the surplus of every macronode in a given period of time does not exceed the sum of the upper bound of arcs leaving that node. To prevent this situation, we introduce dummy arcs unbounded above coming from each supply macronode to the artificial node.

The adoption of dummy arcs leaving surplus macronodes can be interpreted as the assignment of containers to another company or the return of rented containers to trade partners and lessors. Indeed, in their activity carriers are used to offer incentives to give away the surplus of empty containers, because their costs could totally cancel the daily income deriving from the shipment of loaded containers.

At the moment, only depots can cause the problems of infeasibility. Arcs departing from such nodes link other depots and macronodes, or represent the inventory of empty containers held between any given time-period and the subsequent one. All of these arcs have an upper bound due to the reasons heretofore stated. Therefore, we must avoid too many flows reaching the superior capacity. To solve this problem, the usual method is used: the introduction of dummy arcs unbounded above leaving depots and entering the dummy node.

Such arcs could be read as a deficiency in the connections from that depot or in its storage capacity. The cost of these dummy arcs should be set to large numbers, which prevents their selection unless absolutely necessary. Depots are demand nodes in the
last period of temporal horizon, so dummy arcs will be directed towards depots in order to meet these deficits.
Such modifications are shown for a single commodity in Figure 3-3, where dummy arcs have been indicated by a dotted line. Since two types of container are considered in the proposed model, we will adopt two different dummy nodes associated with their own artificial arcs.

3.4 Case study

In this section we aim to verify the suitability of the proposed model to solve potential real world instances. We decided to evaluate its behavior in the context of the Mediterranean basin (Figure 3-4) because, if the economic growth of China and the increase in the value of the Euro continue, this area will shortly need to reallocate an impressive number of empty containers. We assume that in this area ports act as warehouses and some maritime links exist between ports (such connections have been omitted for the sake of clarity in representation).

In previous sections we explained why we chose the hour as time-step in a weekly planning horizon (i.e. 168 time-periods, i.e. the number of hours in a week). In order to define transit times between nodes, we divided their distance by the average speed indicated in Table 3-1. Then transit times are increased by a fixed part taking into account operations performed in depots and macronodes (loading, unloading, etc.).

This table also indicates kilometric costs for each container type and each transportation mode. They have been assessed on the basis of suggestions provided by some experts of the shipping industry. However, the estimation of maritime transportation costs cannot be limited to the data shown in Table 3-1 and needs to be completed by an entry depending on the terminals of departure and arrival. As regards hourly storage costs, we consider values between 45 and 55 cents per 20 ft container, because nowadays daily fees in Mediterranean ports can be estimated on the average at $15 per 20 ft container. We also assume that these costs double for 40 ft containers, because they occupy a double space compared to 20 ft containers.

We assigned the cost of $1000 to dummy arcs associated with the 20 ft containers and the cost of $1500 to artificial variables related to 40 ft containers.
Figure 3-3. The sample multi-period network with dummy nodes and arcs.
Table 3-1. Average speed and transportation costs.

Furthermore, empty container demand and supply have been estimated on the basis of quarterly data presented in the annual report of Mitsui O.S.K. Lines (http://www.mol.co.jp). These data were converted to hourly values for both commodities and then spread over the macronodes of the network.

To solve instances, created in MPS (Mathematical Programming Standard) format, we used two well-known solvers for mixed integer programming, Cplex-mipopt 7.5 (CPLEX optimization, 1995) and Lp_solve 3.2 (Schwab, 1996), running under Red Hat release 2.4.7-10 on a 256 MB 1.7 GHz Pentium IV computer.

The computational efficiency of such solvers is shown in Table 3-2.
New Optimization Models for Empty Container Management

<table>
<thead>
<tr>
<th>Problem</th>
<th>Number of rows</th>
<th>Number of variables</th>
<th>Time for Lp_solve (s)</th>
<th>Time for Cplex (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instance 1</td>
<td>324</td>
<td>809</td>
<td>0.1</td>
<td>0.07</td>
</tr>
<tr>
<td>Instance 2</td>
<td>1150</td>
<td>1652</td>
<td>0.32</td>
<td>0.13</td>
</tr>
<tr>
<td>Instance 3</td>
<td>4158</td>
<td>6658</td>
<td>3.95</td>
<td>0.93</td>
</tr>
<tr>
<td>Instance 4</td>
<td>5658</td>
<td>8992</td>
<td>8.97</td>
<td>1.35</td>
</tr>
<tr>
<td>Instance 5</td>
<td>8631</td>
<td>13957</td>
<td>20.28</td>
<td>2.80</td>
</tr>
<tr>
<td>Instance 6</td>
<td>10593</td>
<td>17237</td>
<td>30.99</td>
<td>3.51</td>
</tr>
<tr>
<td>Instance 7</td>
<td>13239</td>
<td>22205</td>
<td>43.19</td>
<td>4.41</td>
</tr>
<tr>
<td>Instance 8</td>
<td>16695</td>
<td>28487</td>
<td>64.81</td>
<td>6.45</td>
</tr>
<tr>
<td>Instance 9</td>
<td>19677</td>
<td>33469</td>
<td>88.06</td>
<td>8.03</td>
</tr>
<tr>
<td>Instance 10</td>
<td>25143</td>
<td>43081</td>
<td>155.9</td>
<td>12.01</td>
</tr>
<tr>
<td>Instance 11</td>
<td>27629</td>
<td>47395</td>
<td>188.08</td>
<td>11.37</td>
</tr>
<tr>
<td>Instance 12</td>
<td>30272</td>
<td>52030</td>
<td>219.06</td>
<td>12.50</td>
</tr>
<tr>
<td>Instance 13</td>
<td>33103</td>
<td>56699</td>
<td>271.95</td>
<td>17.75</td>
</tr>
<tr>
<td>Instance 14</td>
<td>36429</td>
<td>62687</td>
<td>332.15</td>
<td>20.52</td>
</tr>
<tr>
<td>Instance 15</td>
<td>41905</td>
<td>71992</td>
<td>552.02</td>
<td>26.95</td>
</tr>
</tbody>
</table>

Table 3-2. The performance test of Lp_solve and Cplex.

The commercial solver, Cplex, has shown a good performance in user waiting time. The freeware code, Lp_solve, has proved to be efficient for problems of small and medium size, but user waiting times increase significantly for large instances. In any case the adoption of a hourly time-period in a weekly temporal horizon has proved to be computationally practicable.

Hereunder we provide some comments on the quality of solutions of some instances. As stated previously, the most important elements in the behavior of the proposed model are dummy nodes and arcs, indeed they indicate special problems in fleet management which can be solved by acquiring additional equipment in deficit conditions or moving containers to other companies in surplus situations.

For the sake of simplicity, we consider a network having only one port and one macronode, connected by a pair of opposed arcs. We also assume that the supply of 20 ft containers is by far greater than demand over the planning horizon and vice versa for 40 ft containers (the possibility of substituting one commodity for another has not been taken into consideration). The supply of empty containers in port at the beginning and end of the planning horizon was defined. Then the instance was solved, resulting in a value of the objective function amounting to $2,432,340.

Table 3-3 and Table 3-4 show the values of demand (negative) and supply (positive) in the considered macronode for the different hours of the temporal horizon and illustrate
which deficits and surpluses can be managed using dummy arcs. Table 3-5 reports the
trend of empty containers stored in the port over the planning horizon for both commodities.

Table 3-3. Decisions about 20ft containers over the planning horizon.

Table 3-4. Decisions about 40ft containers over the planning horizon.
Table 3-5. The trend of inventories in the port over the planning horizon.
The first relevant result in this instance is that the selection of dummy arcs leaving the port never takes place over the planning horizon, either for 20 ft or 40 ft containers. As expected in the case of 20 ft containers, given the relevant surplus of such containers in the planning horizon, Table 3-3 shows a frequent selection of dummy arcs leaving the macronode. Usually the model has no special difficulty in meeting deficits. Only the initial part of the planning horizon represents an exception, because sometimes the macronode cannot be reached in time by the port. As a consequence, solvers must exploit dummy arcs to meet such deficits. These flows can represent the number of containers to be leased or purchased, or the possibility of paying a penalty for a failed delivery of the required containers.

Tables 3-3 and 3-5 also show that in the first part of the planning horizon 20 ft containers supplied by the macronode are constantly moved to the dummy node, while deficits are met by inventories available in port, which tend to decrease. In the central part, which is more or less between periods 73 and 93, there is a shortage of 20 ft containers in port. Therefore, the supply of empties at the macronode is not moved to the dummy node, but it is exploited to meet a part of deficits in the same macronode. In the third part of the planning horizon empty containers are no longer moved from the macronode to the dummy node. The supply and demand trend in the same macronode causes, bearing in mind the greater availability of 20 ft containers over demand, a
progressive accumulation of empties in port up to the required number for the last hour of the planning horizon. It is worth noting that at the end of the temporal horizon there is a surplus moving to the dummy node, because such containers cannot reach the port before the conclusion of the same planning horizon.

Moving the focus onto 40 ft commodity, for which a strong prevalence of demand over supply is supposed, Tables 3-4 and 3-5 present two different trends in the first and second phase of the planning horizon. In the first one, excluding the initial deficits that occur when the macronode cannot be reached in time by the port, there is no request for containers met by the dummy node. Demand for empty containers is satisfied by inventories available in the port, that, as a result, tend to decrease. In the second part, deficits are met regularly by the dummy node and empty containers supplied in the macronode cause an accumulation of containers in the port up to the number required for the final hour of the planning horizon.
Chapter 4

A closer overview of shipping companies operations

4.1 Introduction

Tests performed in Chapter 3 have shown that optimization models can significantly support decisions to reposition empty containers properly. Therefore at the moment it is interesting to evaluate the gap between the model previously proposed and daily problems faced by shipping companies. The purpose of this chapter is to present an insight overview of their complex activity, based on information collected through interviews with several experts in the industry. Significant aspects of current logistics practices are highlighted to develop new optimization models addressing the management of empty containers.

The managerial environment of shipping companies is made up of a Head Office and several regional offices throughout the world. The Head Office cares for the maritime transportation service among different regions, designs lines covered by vessels and their schedules. Each local office cares for import and export within a given region to provide door-to-door transportation (or carrier-haulage). As explained by Lopez (2003), shipping companies tend to outsource inland transportation to benefit from the experience and the technical equipment of their partners (trucks, chassis, technical information system, etc.). Shipping companies have sufficient containers to negotiate low rates and good service quality and, if they are not satisfied with their partners performances, they can decide to cooperate with a new company. Basically rail transportation is requested over long distances and high volumes, whereas trucks are supplied for customers that cannot be reached by any other mode.

It is worth noting that carrier-haulage is not compulsory for customers. When it doesn’t meet their needs, they can negotiate separately customized transportation with inland providers and the maritime segment with shipping companies. This option, known as merchant-haulage, forces local agencies to compete with merchants in the domain of intermodal transportation. In the complex environment of shipping, some inland carriers often cooperate with shipping companies to supply carrier-haulage services and, at the same time, compete with them as merchants.
The outline of this chapter is as follows. In section 4.2 major attention is devoted to the relationship between shipping companies and customers. Section 4.3 mainly focuses on the activity performed by shipping companies at a local and global level to meet customers requirements. In section 4.4 some information on rented containers is provided. Finally, section 4.5 illustrates some details on container maintenance and explains the reasons why their status is a major source of uncertainty.

4.2 Export and Import

The activity of shipping companies starts when a potential customer calls the sale office of a local agency. Typically he/she asks for quotations on a given number of containers to be hauled from one port to another. Once the fee of maritime transportation is known, he/she communicates the position of the shipper and asks for the quotation on a possible carrier-haulage service, a door-to-door service including both the maritime segment and land legs. Taking into account the available information, the customer chooses among carrier-haulage, merchant-haulage or looking for different carriers.

When carrier-haulage is chosen, local agencies instruct inland partners about movements to be performed: the location of customers, the pick up time with potential time-windows, the container to be assigned and the departure port. To discourage merchant-haulage, during carrier-haulage empty containers are usually picked up and dropped off in depots closed to customers. At the same time, local agencies force merchants to pick up and drop off empty containers in ports quite distant from customers, involving higher transportation costs for their competitors and more expensive fees for final customers.

Then local agencies set up the list of containers to be loaded on vessels arriving in ports. The Head Office, taking into account the previous list, decides the position of containers on vessels. This is a critical phase because the costs of vessels stopping in ports are rather expensive and unsuitable positions of inbound containers under the deck results in valuable time spent to unload and reload all containers above it.

Moving the focus onto imports, a few days before the arrival of a vessel, the Head Office alerts local agencies operating in the destination port about which loaded containers are to be discharged. As described about exports, inland transportation of inbound containers can be performed in carrier-haulage or merchant-haulage option.
Typically local agencies supply to customers and buy from inland transportation providers three types of carrier-haulage services:

- **Intermodal**, that is a combination of movements performed by trains and trucks coordinated in multimodal facilities. If an intermodal service is planned on a given day, the train will be set up the next day and containers reach the final destination within at least two days after the planning. This service is slow, inexpensive over long distances, and unreliable as well, because it depends on the daily rail schedule.

- **One-way**, performed by trucks. On the one hand, during imports this service ensures loaded containers to be picked up from the destination port and delivered to importers, that can keep containers for few hours. On the other hand, during exports it ensures empty containers for exporters and the transportation of the cargo to the departure port. Usually loaded containers are delivered to exporters in the morning, whereas new cargo is picked up in the afternoon to reach ports in the evening.

- **Round-trip**, performed by trucks. Compared to the one way service, it ensures a high-quality service, because, taking into account requirements specified by receivers, this option allows importers to maintain containers for a longer time (usually some days). Then empty containers are moved by truck to a closed depot indicated by the local agency, waiting for new transportation opportunities.

In order to maximize the profit of shipping companies, local agencies aim to sell two one-ways to customers (one for import and one for export) and buy for this purpose a round-trip service from a trucking company. For instance, when a local agency sells a one-way service at € 320 to a customer, a part of this fee, € 280, is the reward for the inland partner and the remaining € 40 represents the remuneration of the shipping company. On the other hand, when the local agency buys a round trip service at € 480 and sells two one-ways to an importer and an exporter, the profit margins significantly increase: € 320 + € 320 - € 480 = € 160 >> € 80 (the profit deriving from two unmatched one-ways).

This option, known as *street-turn or triangulation*, also enables shipping companies to save handling costs for unprofitable empty containers entering in inland depots.
Furthermore drivers, aware of performing both import and export, are forced to reach importers by noon, in order to pick up the cargo of exporters in the afternoon and move to the departure port in the evening.

Implementing the *street-turn* strategy means dispatching trucks carrying empty containers from importers to close shippers. However, making such decisions does not represent a straightforward task. Typically, when no decision support system is adopted, local agencies determine routes the day before and, because of the large area under their control, this activity is time-consuming. What is more, the planning of routes is an annoying process because sometimes, once one or more itineraries are determined, the set of available information can suddenly change. For instance, due to customers asking to delay the delivery or the collection of goods, or last-minute bookings made by very important customers, the itineraries of trucks, built during several hours of laborious work, becomes ineffective and needs to be set up once again.

Moreover, on the basis of their experience, local agencies must assess the compatibility of goods to be delivered and to be collected. For example, a container importing furniture can be used to serve any kind of export, whereas tank containers have to be cleaned after each delivery and, therefore, need to be moved to a suitable depot (Erera et al., 2005).

It is therefore useful for local agencies to benefit of a decisional tool able to plan the movement of trucks carrying empty containers among importers and suitable exporters. This tool is expected to determine to optimal set of routes within a reasonable computing time, when no new information is expected to arrive. Therefore in Chapter 5 a new mathematical model addressing the street-turn problem is presented. It is applied to the distribution problems of a real-world agency. Although several routing models and algorithms can be proposed for this issue (Laporte, 1997; Toth and Vigo, 2002), due to inherent difficulty of this class of problem, we propose a new approach emphasizing empty flows as decision variables to determine the routes of trucks and serve customers properly.

### 4.3 Fleet management

In order to ensure the continuity of their activity, the so-called street-turn represents the most suitable strategy adopted by local agencies. However, sometimes it cannot be
implemented, because containers need maintenance or there is no booking for the empties returning from importers. As a consequence, some empty containers need to be moved by truck from importers to close depots, waiting for new transportation opportunities. At the same time, other containers are assigned to bookings and dispatched by truck from depots to exporters, according to the standard paradigm of door-to-door service.

In this context, one of the most relevant tasks performed by local agencies is assigning available empty containers stored in depots to customers requests. Typically, three main factors are taken into account by decision-makers in their day-by-day activity: the proximity of vessel departure, the relevance of customers and, finally, the selection of carrier-haulage against merchant-haulage.

In order to maintain enough empty containers to serve customers, local agencies can exploit different options. The first one consists of renting containers, but the Head Office is not willing to spend money in this way. Rather it suggests moving empty containers among inland and maritime depots by railway, balancing the difference in supply and demand. However, in several countries it is not allowed to buy transportation capacity for trains moving just empty containers. They must be put on the same trains as loaded ones. To make matters worse, loaded containers have a greater priority, because they generate profits for shipping companies, and repositioning times for empty containers are often unreliable. As a result, different options must be considered to provide depots with sufficient empty containers.

Sometimes local agencies can resort to substitutions, that is serving customers with a container type different than the requested ones. When substitutions are performed, the customer pays the fee for the requested container, whereas shipping companies bear the remaining part of the total rate. Typically, high cube containers can be used instead of general purpose ones and flat containers can replace the open top ones. Other substitution options are not implemented.

Another relevant strategy is the storage of empty containers in depots in anticipation of future transportation requests. However, the accumulation of too many empties involve unnecessary storage costs, so their inventory level must be carefully checked every day. Since the shortage of empties may result in the risk of not meeting customers requirements, a trade-off must be achieved. Moreover, a proper management needs to
consider the time perspective explicitly. Furthermore, decisions have to be made when there is only a partial knowledge of the problem and the introduction of uncertain parameters should be taken into account as well.

Since local agencies have several options in meeting bookings and ad-hoc decision making is highly inadequate, their activity can significantly benefit from the development of optimization models able to support empty container allocation in the context of a land distribution systems. Taking into account the framework by Crainic et al. (1993-b), in Chapter 6 two optimization models are proposed in order to ensure enough containers available in depots closed to customers, while minimizing inventory and transportation costs. The first model takes into account company-owned containers only. The second one considers rented containers as well, according to logistic practices described in the next paragraph.

When container shortage is expected in the near future and the previous options are not sufficient, local agencies ask the Head Office to allow unloading empty containers from arriving vessels. Nowadays, since local agencies become aware of the possibility of unloading containers one day before the arrival of vessels, they do not rely on this strategy. As a consequence, they will significantly benefit from the development of a reliable dynamic asset management supporting maritime reposition of empty containers among different ports.

Several sources of uncertainty affect maritime reposition, like the number of empty containers requested at ports and the time they arrive at ports. Moreover, since loaded containers have greater priority than empties and unexpected bookings can suddenly arise, the residual capacity for empty containers on vessels is uncertain as well (Cheung and Chen, 1998).

In addition, one of the most relevant factors affecting the low quality of large-scale maritime reposition is the so-called “Cut and Run”. When a vessel berths, reefer and tank containers are loaded first, due to their high storage cost. Then standard loaded containers are put on vessels. Finally, empty containers are loaded. If vessels have delays in their schedule, local agencies, who take care of terminal operations on behalf of the Head Office, ask for a “Cut and Run”, that is the conclusion of operations for vessels, that will leave ports without loading empty containers. For instance, due to adverse climatic conditions, some ports interrupt their activity and, when they restart
their operations, local agencies “cut” empty containers to gain time. Typically a “cut container” will be assigned to the following vessel scheduled in that line. It is worth noting that “Cut and Run” practices are usually caused by local agencies when they define the berthing time of vessels, because they take into account loaded containers mainly.

Furthermore, ports may impose different rules regarding empty containers. One the one hand, some large ports have enough space to dedicate a specific area to store empties. On the other hand, some small ports do not have an area for this type of equipment and require outbound empty containers to be assigned to a set of vessels (usually two or three) before their arrival from the land side. In the first case decision-makers can decide tomorrow about empty containers arriving from the landside in the following 24 hours from now. In the second case they must decide immediately the most suitable vessel for the uncertain number of empty containers that may reach ports from the landside in the following 24 hours.

When empty containers must be assigned to vessels before their arrival from the landside, local agencies tend to make decisions about reposition. Typically, agencies adopt the rule of assigning empty containers to vessels moving towards export-dominant areas (for instance the Far East) or empties in a hub, where they can be stored. As a consequence, this practice is not able to see containers requested by other agencies. Fleet management will be significantly improved if the Head Office plan maritime reposition, taking into account information provided by different local agencies about surplus and deficit of empty containers.

To conclude, a central control of maritime reposition can surely support the activity of local agencies. In Chapter 7, a new mathematical model is presented to face this issue.

4.4 Rented containers

Shipping company fleets are made up of owned, leased and rented containers. Leasing companies allow shipping companies to obtain the property of containers by paying a long-term rental fee for a given period of time (usually a few years). According to Foxcroft (2004), this rate attained US$ 0.85 per day for a 20ft general purpose container.
Regarding rented containers, two main options exist. On the one hand, it is compulsory that some of them are returned after the delivery at final destination. On the other hand, the so-called flexible containers can be off-hired or reused by local agencies, depending on their needs.

While leased containers can be modelled as company owned containers, flexible containers offer a different array of decisions. In order to address their management properly, we focus onto contracts stipulated by shipping companies and lessors. One the one hand, shipping companies prefer renting containers in regions which have a shortage (for example the Far-East) and returning them to regions which have surplus (for example the West Coast of USA). On the other hand, lessors, that supply containers when shipping companies have a need for them, are not satisfied with receiving containers in import-dominant regions, because they have to bear relevant reposition costs to bring containers where they may be requested.

In this complex environment, lessors specify in contracts a per-diem rate and the maximum number of containers that can be off-hired in each depot per month. Moreover they impose drop-off fees when containers are returned. Such fees are particularly expensive in surplus regions, where reduced transportation opportunities make empty containers useless. Pick-up fees are applied when containers are rented in deficit areas, where empties are valuable, because significant requests occurs. Since surpluses and deficits take place in different parts of the world, shipping companies, taking into account pick up and drop off fees, must decide if flexible containers will be repositioned in deficit regions or returned to lessors.

Nowadays shipping companies adopt logistic solutions, that mainly depend on their business volume. On the one hand, during peak seasons they tend to benefit from rented containers as opposed of buying new ones. Therefore, they usually keep and reposition rented containers to avoid high pick-up charges in export-dominant regions, such as the Far-East. On the other hand, during slack seasons, the Head Office force agencies to reduce their fleet, and flexible containers are returned to their owners. Such decisions save high repositioning and storage costs, but high drop-off costs in import-dominant regions must be paid (Hanh, 2003).

Moving the focus onto the daily activity of shipping companies, once a vessel berths in a port, loaded containers to be off-hired are identified using documentation. Then
containers are dispatched to the final destination through compulsory round trip services to return empty containers in depots identified by lessors. It is therefore crucial for shipping companies and their local agencies to manage flexible containers properly. They need to be assisted by a decisional tool able to address the decisions of repositioning flexible containers or returning such an equipment, bearing drop off charges. This tool is expected to determine solutions within a reasonable computing time. The second model proposed in Chapter 6 will call for such decisions taking into account a heterogeneous fleet made up of company-owned and flexible containers.

4.5 Logistic, maintenance and uncertainty

Containers last more or less 10 years, depending on their type and the frequency of use. During their life, they are repeatedly moved from one part to another part of the world, loaded, unloaded, put and discharged on vehicles. As a consequence, they often need maintenance, repair and cleaning, so that they can be provided to customers in good conditions. A potential refusal of a container involves relevant logistic costs, that is the cost of two empty trips for the first container and the cost of providing a new one. Moreover, container rejection represents a negative point for the appeal of shipping companies and exposes the risk of loosing customers. However, it is not possible to define a standard for a “satisfactory” container, because customers have different requirements. As a result, shipping companies must often check the status of their equipment and perform their repair. This process introduces significant sources of uncertainty on their availability.

As explained in Chapter 2, when a container is transferred from the liability of an operator to another, both parties undertake an inspection and the so-called Equipment Interchange Receipt document is set up to attest the transfer of containers from one operator to another and indicate their conditions at that moment. This document highlights the following class of damages:

- impact;
- cleaning;
- wear and tear;
- improper repairs.
Interchanges allow to state possible damages and find out the responsible operator, who will bear the impact, cleaning and improper repair costs, whereas wear and tear damages are not taken into account. For instance, customers might be charged by shipping companies because of the damage involved by spillage of special products on the interior surface of containers. In these cases customers refund carriers taking into account the costs of repairing containers and moving scrap to specialized sites, according to strict environmental laws.

When wear and tear damages become relevant, local agencies must decide if containers will be repaired or, if it is possible, off-hired. Sometimes damaged containers are bought by leasing companies and lessors, who take care of maintenance costs and reuse the equipment for their business. On the other hand, a lower standard of repair is regularly adopted when containers will stay under the control of shipping companies, so that a typical customer won’t refuse this equipment.

The need to perform maintenance operation introduces uncertainty on container availability. At the time of the decision reliable information of container status occurs only for the initial part of the planning horizon and adequate statistics are usually unavailable. As a consequence, local agencies are forced to make decisions without having a certain perspective of empties that may be offered to customers in future periods. In Chapter 6 this uncertainty will be modelled implementing decisions in a rolling horizon fashion, whereas in Chapter 7 scenario analysis is adopted. Further details about these methods are available in Powell et al. (2005).

Uncertainty on container availability occurs during street-turns as well, because the allocation of containers among importers and exporters is planned before having complete information of their status. Such a situation results in the risk of providing containers having unsuitable conditions for customer requirements. In this context, drivers represent the only people able to check the conditions of empty containers properly and, on the basis of their own experience, they can decide if they may be accepted by exporters. If containers are a bit dirty, they are also requested to carry out the cleaning quickly. According to the driver’s opinion, when containers previously assigned to exports need immediate maintenance, local agencies have to quickly correct the set of routes to be assigned to trucks, so that all shippers receive equipment as requested.
Therefore, it is useful for local agencies to adopt a decisional tool, able to plan the itineraries of trucks carrying empty containers among importers and suitable exporters. This tool is requested to determine the optimal set of routes the day before within a reasonable time, when no new information is expected to arrive. What is more, such a characteristic enables this tool to adjust routes quickly when unexpected events occur, such as failures, road accidents, etc.
Chapter 5

An optimization model for the street-turn problem

5.1 Introduction

Street-turn plays a significant role for local agencies in order to achieve high performance level in the management of their equipment. This option is based on allocating an empty container by truck from an importer to an exporter without first returning to a port. In a perfect world, an exporter would also be an importer and commodities imported and exported could be moved in the same type of container. However, this situation rarely occurs and local agencies have to reposition their empty containers to meet future transportation opportunities.

The problem of assigning imported containers for local export in the trucking industry has been investigated in the context of the Vehicle Routing Problem with Backhauls (Golden et al., 1985; Casco et al., 1988). It is a major variant of the so-called Vehicle Routing Problem, one of the most relevant problems arising in the field of combinatorial optimization (Laporte, 1997; Toth and Vigo, 2002). It requires the determination of the optimal routes to be performed by a fleet of vehicles stationed at a common depot to serve customers, while satisfying several operational constraints. However, all variants of the Vehicle Routing Problem are well-known for their intrinsic difficulty and large instances cannot be quickly solved by exact algorithms. Several heuristic and metaheuristic methods have been developed to yield effective solutions, even if the optimality of distribution plans is not ensured (Laporte and Semet, 2002; Gendreau et al., 2002).

In this chapter we present an approach to the street-turn problem different from standard routing formulations, which calls for a set of circuits covered by trucks through a single mathematical model. We propose an optimization model that solves the allocation of empty containers among importers and exporters and provides useful information on the routes of trucks, when they are stationed at a common port. The circuit associated with each route is completely identified by links directed from the port to importers, from importers to exporters according to the model solution, and from exporters to the port. The most important advantage of this approach is the opportunity of dealing with the
empty container allocation issue, that can be solved by exact algorithms more easily than a routing problem (Olivo et al., 2005). Moreover this study is also motivated by the relevant opportunity to solve effectively the street-turn problem on behalf of a real-world local agency. We compare its decisions to analytical solutions provided by the proposed model. We aim to show that this optimization model provides better results in terms of both total travel distance and time requested to set up the routes of trucks.

Moving the focus onto the scientific literature in the field of Vehicle Routing Problem with Backhauls, it is widely accepted that distribution problems, taking into account both importers and exporters, can result in significant savings in transportation costs (Cundill and Hull, 1979; Dart, 1983; Jordan and Burns, 1984). Deif and Bodin (1984) proposed a heuristic method that, moving from the infeasible solution where each customer is served by a route, merges such circuits on the basis of potential savings. To get high-quality results, they delay the combination of the mixed routes by penalizing links among different customer types. Goetschalckx and Jacobs-Blecha (1989) presented a heuristic approach that separately builds routes among importers and exporters. Then such routes are combined in the final collection of itineraries. However, both previous algorithms do not permit to control the final number of routes, thus often resulting in distribution plans to be rejected.

Anily (1996) computed a lower bound on the optimal total cost and adopted a heuristic algorithm to find a feasible solution. He showed that the complexity of such an approach depends mostly on the size of customer requests. Toth and Vigo (1997) reformulated the Vehicle Routing Problem with Backhauls as an asymmetric problem. They proposed a branch and bound method computing lagrangian lower bounds. Mingozzi et al. (1999) presented a set partitioning formulation using binary variables, each associated with a different feasible route. They computed effective lower bounds by adopting several heuristics to solve the dual of the lagrangian relaxation of the original model. Toth and Vigo (1999) considered sets of customers containing only importers or exporters. Then such clusters are merged by exploiting the information associated with good lower bounds computed through a lagrangian relaxation. Then a heuristic based on local search was adopted to obtain fast and feasible solutions.
An approach different from standard routing models was adopted by Jula et al. (2006), that investigated the pattern of empty container flows in the port complex of Los Angeles and Long Beach. In order to reduce the impressive number of empty trips around maritime terminals and the resulting environmental problems, they developed an operational dynamic model to minimize the cost of container reuse, taking into account both street-turn and depot-direct options. According to the last strategy, empty containers can be stored in inland depot as well, but it is not adopted by the local agency of this case study. Moreover the opportunity to operate in new depots is a complex topic arising at a strategic planning level that cannot be properly evaluated by operational models (Crainic et al., 1989).

The outline of this chapter is as follows. In section 5.2 the street-turn problem is represented through an integer programming model. In section 5.3 the real case study is presented. Some instances are solved and compared to decisions made by a local agency. Section 5.4 presents a summary of conclusions and describes future research in the field of the street-turn issue.

5.2 Optimization model

The purpose of this paragraph is to present a mathematical model that can be adopted by a local agency to properly dispatch its empty containers among customers and determine the routes of trucks as well. This agency outsources inland transportation to a trucking company, that must be instructed on movements to be performed. All routes covered by trucks start and end in a given port denoted by letter $p$.

The distribution of a single container type, the 20’ general purpose, is considered. The trucking company adopts identical trucks, each carrying two 20’ containers at most. A set $D$ of exporters and a set $S$ of importers are considered. Each importer $i \in S$ supplies a number $s_i > 0$ of 20’ empty containers that can be reused for exports. Each exporter $i \in D$ demands a number $d_i > 0$ of 20’ empty containers. Since the number of available containers is usually different than the equipment booked, in order to find feasible solutions, the port $p$ provides empty containers to exporters in deficit conditions and receive such an equipment from importers when a surplus occurs. To clarify, the port $p$ supplies or requires $-\sum_{i \in S} s_i + \sum_{i \in D} d_i$ empty containers, so that the network is balanced.
The problem is presented as an integer programming model, that incorporates both the allocation of containers and the routing of trucks. Two classes of variables are considered for this purpose, denoted by letters $x$ and $y$ respectively:

- Variable $x_{ij}$ denotes the number of empty containers hauled from the importer $i \in S$ to the exporter $j \in D$.
- Variable $x'_{ij}$ indicates the number of empty containers hauled from the importer $i \in S$ to the importer $j \in S$.
- Variable $x''_{ij}$ denotes the number of empty containers hauled from the exporter $i \in D$ to the exporter $j \in D$.
- Variable $x_{ip}$ indicates the number of empty containers allocated from the port $p$ to the exporter $i \in D$. Such a variable takes positive values when the number of containers requested by exporters exceeds the number of containers provided by exporters. As a consequence, local agencies need to resort to empty containers stored in port $p$ to serve exporters.
- Variable $x_{ip}$ denotes the number of empty containers allocated from the importer $i \in S$ to the port $p$. Such a variable takes positive values when the number of containers requested by exporters is lower than the number of containers provided by exporters. As a result, local agencies need to move the surplus of empty containers to the port.

Moving the focus onto trucks, the following set of variables is defined:

- Variable $y_{ij}$ denotes the number of trucks, each carrying at least one empty container, moved from the importer $i \in S$ to the exporter $j \in D$; $c_{ij}$ represents the related unitary cost.
- Variable $y'_{ij}$ indicates the number of trucks, each carrying at least one empty container, moved from the importer $i \in S$ to the importer $j \in S$; $c'_{ij}$ represents the related unitary cost.
- Variable $y''_{ij}$ denotes the number of trucks, each carrying at least one empty container, moved from the exporter $i \in D$ to the exporter $j \in D$; $c''_{ij}$ represents the related unitary cost.
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- Variable $y_{pi}$ indicates the number of trucks, each carrying at least one empty container, moved from the port $p$ to the exporter $i \in D$; $c_{pi}$ represents the related unitary cost.
- Variable $y_{ip}$ denotes the number of trucks, each carrying at least one empty container, moved from the importer $i \in S$ to the port $p$; $c_{ip}$ represents the related unitary cost.

The optimization model can be expressed as follows:

$$\min \sum_{i \in S} \left( \sum_{j \in D} c_{ij} y_{ij} + \sum_{j \in D} c_{ji} y_{ji} + c_{pi} y_{pi} \right) + \sum_{i \in D} \left( \sum_{j \in D} c_{ij} y_{ij} + c_{pi} y_{pi} \right)$$

(subject to)

$$\sum_{j \in S} \left( x_{ij}^\prime - x_{ji}^\prime \right) + \sum_{j \in D} x_{ij}^\prime + x_{ip} = s_i \quad \forall i \in S$$

$$\sum_{j \in D} \left( x_{ij}^\prime - x_{ji}^\prime \right) - \sum_{j \in S} x_{ij}^\prime - x_{pi} = -d_i \quad \forall i \in D$$

$$\sum_{i \in S} x_{ip} - \sum_{i \in S} x_{ip} = -\sum s_i + \sum d_i$$

$$x_{ij}^\prime \leq 2 y_{ij} \quad \forall i \in S, \forall j \in D$$

$$x_{ij}^\prime \leq 2 y_{ij} \quad \forall i, j \in S$$

$$x_{ij}^\prime \leq 2 y_{ij} \quad \forall i, j \in D$$

$$x_{ip} \leq 2 y_{ip} \quad \forall i \in S$$

$$x_{pi} \leq 2 y_{pi} \quad \forall i \in D$$

where all variables can take only non-negative integer values.

The objective function (5.1) minimizes the transportation costs of trucks carrying at least one empty container. Constraint set (5.2) represents the mass balance conservation of empty containers for each importer $i \in S$. It requires that all containers returned by importers are picked up. Constraint set (5.3) represents the mass balance conservation of empty containers for each exporter $i \in D$. It requires that all exporters receive the requested number of empty containers. Constraint sets (5.4) ensures flow conservation of empty containers in the port $p$. Constraints (5.5), (5.6), (5.7), (5.8) and (5.9) guarantee that each truck does not carry more than two empty containers for each type of link.

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Figure 5-1 presents a sample instance of the problem. Continuous lines represent decision variables determined by the proposed model. However, in order to call for the complete circuit covered by each truck, additional links, represented by dotted lines, are introduced among the port and importers and among exporters and the port.

5.3 Case study

Early tests were performed considering as numerical value of costs the related shortest road distance, because the comparison among the proposed solution and decisions made by a real trucking company will be made on the basis of this parameter. However, early results were not so valuable because unsuitable routes were often determined. Then tests were performed by adopting the average travel time among nodes instead of the road
distance, because, according to their daily logistic practices, trucking companies prefer the fastest routes over the shortest ones. In this case, at times the optimization model provides good distribution plans. However, more often it suggests using more trucks than needed, because it does not take into account loaded movements. To clarify, a graphical example of this unsuitable circumstance is proposed in the case 1 of Figure 5-2. It shows an importer supplying two empty containers to a pair of exporters, each asking one container. The distribution plan returned by the optimization model indicates using two different trucks to serve exporters. A more suitable solution may be produced moving a single truck to serve both customers, as shown in case 2.

Figure 5-2. Drawbacks in distribution plans (Case 1) and desiderate solutions (Case 2).
In order to get the distribution plan shown in case 2, links among exporters should be encouraged, because they result in the use of a reduced number of trucks and involve significant savings for trucking companies. In this context, it is worth noting that transportation, among exporters $i \in D$ and $j \in D$, should take into account transit times among these customers. Moreover, it is always followed by a truck movement from exporter $j$ to the port $p$. What is more, a potential link, among exporters $i \in D$ and $j \in D$, would save the cost of moving a truck with a single loaded container from the first exporter $i$ to the port $p$.

Denoting by $t_{ij}$ the transportation time among customers $i$ and $j$, the following formulation is then adopted:

$$ c_{ij} = t_{ij} + t_{jp} - t_{ip} \quad \forall i, j \in D $$

(10)

The link from importer $i \in S$ to importer $j \in S$ is always preceded by a truck movement from the port $p$ to $i \in S$. Moreover, the link from $i \in S$ to $j \in S$ would save the cost of dispatching a truck with a single loaded container from $p$ to serve the importer $j \in S$. In this case, the following formulation is adopted:

$$ c_{ij} = t_{pj} + t_{ip} - t_{pj} \quad \forall i, j \in S $$

(11)

The link from importer $i \in S$ to exporter $j \in D$ should also take into account the transportation time from the port to the importer and from the exporter to the port:

$$ c_{ij} = t_{pi} + t_{pj} + t_{jp} \quad \forall i \in S, \forall j \in D $$

(12)

Finally, the transportation from importers and to the port and from the port to exporters are penalized, because they involve a round trip movement, one for the empty containers and one for loaded flows. As a consequence, the following formulation cost is adopted:

$$ c_{ip} = t_{pi} + t_{ip} \quad \forall i \in S $$

(13)

$$ c_{pi} = t_{pi} + t_{ip} \quad \forall i \in D $$

(14)

The cost formulations (10), (11), (12), (13) and (14) are able to account for the whole travel time for trucks covering a route starting and finishing in the port $p$. In what follows, we show the potential cases that can occur:

- A vehicle serving one importer $i \in S$ and two exporters $j, k \in D$:

$$ c_{ij} + c_{jk} = t_{pi} + t_{ij} + t_{jp} + t_{jk} + t_{kp} - t_{jp} = t_{pi} + t_{ij} + t_{jk} + t_{kp} $$

- A vehicle serving two importers $i, j \in S$ and one exporter $k \in D$:
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\[ c_{ij}^c + c_{jk}^c = t_{ij} + t_{pi} - t_{pj} + t_{jp} + t_{jk} + t_{kp} = t_{pi} + t_{ij} + t_{jk} + t_{kp} \]

- A vehicle serving two importers \( i, j \in S \) and two exporters \( k, w \in D \):

\[ c_{ij}^c + c_{jk}^c + c_{kw}^c = t_{ij} + t_{pi} - t_{pj} + t_{jp} + t_{jk} + t_{kp} + t_{kw} + t_{wp} - t_{kp} = t_{pi} + t_{ij} + t_{jk} + t_{kp} + t_{wp} \]

- A vehicle leaving the port \( p \) to serve two exporters \( i, j \in D \):

\[ c_{pi}^c + c_{ij}^c = t_{pi} + t_{ip} + t_{ij} + t_{jp} - t_{ip} = t_{pi} + t_{ij} + t_{jp} \]

- A vehicle leaving two importers \( i, j \in S \) to reach the port \( p \):

\[ c_{ij}^c + c_{jp}^c = t_{ij} + t_{pi} - t_{pj} + t_{pj} + t_{jp} = t_{pi} + t_{ij} + t_{jp} \]

At first such cost formulations were adopted and validated in the context of several artificial instances. Then major attention was devoted to the set of daily instances provided by a local agency. Once determined the variables of the problem and the itineraries to be covered by trucks, the length of every route was compared to the total distance travelled by the fleet, according to decisions made by the local agency in more than four hours.

Table 5-1 shows some parameters on a generic instance provided by the local agency. To solve this instance, the well-known solver Cplex-mipopt 7.5 (CPLEX optimization, 1995), running under Red Hat release 2.4.7-10 on a 256MB 1.7 GHz Pentium IV computer, is adopted. Rows indicate the number of customers, the number of decision variables, the user waiting-time to get the optimal distribution plan and the potential savings that could be achieved, taking into account the routes determined by the local agency.

<table>
<thead>
<tr>
<th>Number of importers</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of exporters</td>
<td>29</td>
</tr>
<tr>
<td>Number of variables</td>
<td>2634</td>
</tr>
<tr>
<td>Time for Cplex (s)</td>
<td>311.96</td>
</tr>
<tr>
<td>Local agency decisions [Km]</td>
<td>10411</td>
</tr>
<tr>
<td>Analytical distribution plan [km]</td>
<td>9985</td>
</tr>
<tr>
<td>Saving</td>
<td>4%</td>
</tr>
</tbody>
</table>

Table 5-1. Solution of the numerical problem.

Previous table shows that Cplex converges to the optimum within reasonable computational times fitting the practical needs of the local agency. Furthermore, the hard work saved for decision-makers to determine routes represents one of the most
important benefits resulting from the proposed approach. The human brain indeed is not capable of managing such a multitude of information and, therefore, cannot ensure a fast and efficient solution for such a complex problem. Moving the focus onto the distribution plans, previous results indicate that significant savings can be achieved in terms of total travelled distances.

Finally, it is worth noting that the local agency checked the routes determined through the proposed approach and attested their suitability for its distribution problems.

5.4 Conclusions and future developments

The proposed approach represents a tool able to support the decisions of local agencies about street-turns. By emphasizing the network structure of the empty container allocation issue, this contribution proposes an optimization model to determine the routes of trucks. An exact algorithm was implemented and distribution plans are determined within practical user waiting times. The comparison among such plans and decisions made by a real local agency indicates that this tool represents a promising instrument to improve their decisional skills.

At the moment, valuable work still needs to compare the proposed optimization model with a classical Vehicle Routing Problem with Backhauls formulation, in terms of both computing times and solution effectiveness. Moreover, to solve the street-turn issue in a larger set of ports, we are going to propose a Multi-depot Vehicle Routing Problem with Backhauls formulation and consider potential time window restrictions.

Research is still in progress in modellistic aspects to propose a dynamic version of the proposed formulation. The time-perspective will be able to select which customers should be served in each period of the planning horizon, whereas, at the moment, the set of customers to be considered on a given day is provided \textit{a priori} by the local agency. However, sometimes in a dynamic environment decisions have to be made when there is only a partial knowledge of the problem parameters. As a consequence, a stochastic formulation will be proposed as well.

Since dynamic and stochastic formulations result in large instances to be solved within reasonable and suitable computing times, algorithmic developments should be proposed. For example, since strong algebraic structures may arise in such models, they
can be exploited resorting to decomposition algorithms, lagrangian relaxations and structured interior point methods.
Chapter 6

Two optimization models for the local reposition of empty containers

6.1 Introduction

As described in the previous chapter, the favorite strategy adopted by local agencies to serve customers is the so-called street-turn. However, sometimes empty containers cannot be directly moved from importers to exporters, because they need maintenance or there is no request for the equipment returning from recipients. As a consequence, some empty containers need to be moved by truck from importers to the closest depots, waiting for new transportation opportunities. At the same time, other empty containers are dispatched by truck from depots to exporters, according to the standard paradigm of door-to-door service.

Since in each depot the number of containers returning after imports and requested for exports is normally not the same, local agencies have to plan the reposition of their empties among the different storage areas under their control. This day-by-day issue represents a decisive factor for the competitiveness of shipping companies, because local agencies must maintain sufficient containers in depots over time to satisfy customers requests. Failure to provide enough empties results in the risk of competitors supplying containers as requested and, in a highly competitive market, some transportation opportunities may be lost.

In order to ensure enough containers for customers requests, local agencies can resort to different options. The most apparent consists of moving, typically by rail, empty containers among maritime and inland depots. Although such movements do not produce immediate revenues, they allow empties to be properly positioned when new bookings will arise. However, sometimes this strategy is not effective because profitable loaded containers receive greater attention than empties. Moreover, usually specific national rules do not allow setting up trains carrying empty containers only. It results in a limited transportation capacity and unreliable reposition times. As a consequence, other options have to be considered to supply reliable transportation services to customers.
A possible alternative strategy is represented by substitutions, that is serving customers with a container type different than the requested ones. However, substitutions imply operating costs for local agencies, because, while customers pay a fee for moving the requested container, shipping companies bear the remaining part of the total rate.

Another strategy is based on the accumulation of empty containers in depots in anticipation of future transportation requests. However, too many empty containers held in depots involve unnecessary storage costs, so their inventory level must be carefully checked. As a result, a proper management of local agencies needs to take into account the time perspective explicitly, to represent decision outcomes and select the most suitable ones.

The opportunity of incorporating in current decisions their impact on system future is conflict with the need to make decisions when there is only a partial knowledge of the problem parameters. Therefore, some assumptions on the system evolution have to be made about the sources of uncertainty affecting the local management of empty containers. One of the most relevant sources of uncertainty is probably containers availability, because an inspection is performed to verify their effective status, when they are transferred from one part to another. Since shipping companies have to provide shippers with containers in good condition and their status cannot be known before the check, local agencies do not have a priori information on the time they will become available. To make matters worse, when importers buy a round trip service, containers can be held by customers for several days and it is not possible to know when they will be returned to serve future requests.

It is worth noting that bookings do not represent such a relevant source of uncertainty, because customers tend to book containers some days in advance and the majority of orders are predictable. Nevertheless, sometimes unexpected transportation requests from important customers arrive and, since they cannot be refused, local agencies are forced to satisfy their needs. Finally, delays and equipment failures represent uncertain events adding complexity to fleet management, because they may have serious impacts on the quality of service provided to customers.

Two optimization models are proposed to deal with the local reposition of empty containers over a dynamic environment. A heterogeneous fleet of container is considered and, since their different sizes result in different use of storage spaces, a new
set of mutual capacity constraints is proposed. Substitution options are taken into account as well, resulting in a mathematical model able to address a wide array of decisions. Moreover both models exhibit some strong algebraic structures that can be exploited to develop specialized algorithmic approaches.

The outline of this chapter is as follows. In paragraph 6.2 the most significant literature in the field is revised. Paragraph 6.3 presents the optimization model adopted for a heterogeneous fleet of company-owned containers. In paragraph 6.4 several instances are solved by a standard solver for mixed-integer programming. Paragraph 6.5 introduces an improved mathematical model that, taking into account both owned and rented containers, results in a more complex formulation than previous ones. In paragraph 6.6 several instances are solved by different solvers to reach the optimum within adequate computing times. Finally, paragraph 6.7 illustrates some attractive research perspectives in the field either from the modellistic and algorithmic point of view.

6.2 Literature review

Although the problems of local agencies are typically thought of as decisions to move containers from one place to another, daily problems involve a large array of choices such as renting/selling, repairing, cleaning and storing equipment and, whenever possible, substituting one container type for another. Given the wide breadth of activities planned by shipping companies, ad hoc decision-making processes seem highly inadequate. As a result, several authors have faced this problem using Operation Research methods to enhance the analysis of distribution planning and achieve the best trade-off between operating costs and high quality services. Moreover limited work has been conducted in the context of land distribution systems, whereas many studies have focused on the maritime reposition issue.

Dejax and Crainic (1987) reviewed the most significant papers on the management of empty flows. They stated that, although this problem had received much attention, poor consideration had been dedicated to the development of original models addressing the allocation of empty containers in the context of land distribution systems. They mentioned few authors investigating the reposition of empty containers from surplus
ports to shortage ports, using both network and linear programming formulations in a deterministic dynamic environment.

Crainic, et al. (1989) discussed the strategic issue of assigning customers to depots in an inland transportation network managed by a shipping company. They proposed an optimization model to minimize the cost of depot opening and empty container transportation. The solution of their formulation represents the starting point for the operational models presented in this chapter of the thesis.

Crainic, et al. (1993-b) presented a general framework to address the specific characteristics of the empty container allocation problem in a land distribution system of a shipping company. They developed two deterministic dynamic models for the single commodity case and the multicommodity variance. No computational result was provided. Moreover a small number algorithms has been proposed and validated (Abrance et al., 1999) to solve their formulations.

Shen and Khoong (1995), focusing on the business perspective of the shipping industry, developed a decision supporting system for the maritime reposition of empty containers in the single-commodity case. They minimized repositioning costs and provided decisions covering leasing and returning a from external sources. Their contribution was based on a network optimization model because several efficient algorithms can solve this type of formulation. However, no computational result was provided.

Since in the real world it is necessary to deal with stochastic systems, Cheung and Chen (1998) applied the network recourse formulation to the dynamic maritime reposition of empty containers. Although shipping companies typically manage different container types, the authors considered the single-commodity case.

Holmberg et al. (1998) investigated the distribution planning of empty cars in a railway company. To reduce the shortcomings of the existing planning process, they proposed a dynamic multicommodity network, taking into account transportation capacities constraints for empty vehicles and the exact timetables. Substitution possibilities were not included.

Jiele (1999) proposed a deterministic dynamic model for the maritime reposition of empty containers. He adopted a minimum cost flow algorithm for the single commodity case and a linear programming technique for the multicommodity variance. In the latter
case, the author considered two container types having the same size. Substitution options were not taken into account.

Choong, et al. (2002) investigated the end-of-horizon effect on the management of empty containers in a land distribution system. Some of their assumptions are relaxed in this thesis. First of all, they took into account a single container type and, as a result, substitution options were not included. Moreover company-owned containers were modelled in the same way as rented containers. Finally they assumed that empty containers can be rented in any time-period.

Olivo et al. (2005) proposed a deterministic dynamic optimization model to support empty container reposition in a multimodal network. They considered both maritime and inland distribution in a continental scale. However, taking into account the logistic practices adopted by shipping companies, inland allocation and maritime reposition should be addressed by different optimization models. Furthermore, although the authors considered two container types, substitution options were included. Finally they modelled rented containers as company-owned containers.

Erera et al. (2005) proposed a deterministic dynamic large-scale optimization model to simultaneously manage loaded and empty containers for the needs of a tank container operator. They took into account both maritime and inland distribution in a continental scale. A single container type is considered and, as a consequence, substitution options are not included.

6.3 The first optimization model

In order to address properly empty container management from the point of view of a local agency, the proposed model assumes as decision variables the number of empties to be moved among the different storage areas, to be stored in depots and identifies potential substitutions among different container types. This model aims to minimize the operational costs resulting from such decisions in a dynamic environment.

In order to model customer needs, the demand of empty containers, that is known by local agencies through their booking desks, is associated with a suitable inland depot, that is expected to provide empties as requested. For instance, the model proposed by Crainic et al. (1989) can be adopted to assign customers to depots. Each request must be satisfied in a suitable period of time, such that loaded containers can reach vessels in
time. At the same time, other containers returning from importers are moved to close inland depots, where they are checked to verify their status. Typically some of them have good conditions and can be stored, waiting for new transportation opportunities. On the other hand, some containers need maintenance, so they are repaired, cleaned and finally stored. In this context, inland depots are requested to inform local agencies daily about the correct number of empty containers stored, clean and available that can be assigned to customers.

It is worth noting that the number of empty containers stored in ports does not represent a decision variable for local agencies, because the maritime reposition represents a specific task for the Head Office. Nevertheless, ports play an important role in the local reposition issue, because the number of loaded containers imported and exported in the region is generally not the same and empty containers can be either in surplus or deficit. When surpluses take place in a region controlled by a local agency, empty containers are moved to ports and supplied in the context of the maritime reposition issue. When the regional deficit occurs, ports are requested to provide an adequate number of empty containers to serve exporters.

The information set previously described can be presented through network flow models, indeed the need to satisfy customer requests and use available containers can be expressed in terms of mass conservation constraints (Ahuja et al., 1993). Sometimes networks are generated in a space-time perspective, to determine the best decisions for the current time and assess their impact on future periods. Often the day is adopted as time-step over a standard fifteen periods planning horizon.

A relevant difficulty with such a multi-period formulation is the uncertainty of some parameters, such as the number of bookings and the time when containers become available for export. In this specific application, we assume to know the state of the system and its evolution over time, so the resulting optimization model is deterministic. In this context, while a part of parameters is known, some others represent a realization of uncertain parameters.

Once uncertain parameters are observed, the initial policies have to be corrected. Such a phase, that is known as recourse formulation, is performed in this specific case implementing decisions in a rolling horizon fashion. It means that, as time passes, new information becomes available, a new period is added at the end of the planning horizon.
and the problem is solved again according to updated data. To clarify, instances are solved using, as information available at the beginning of the planning horizon, the certain number of containers supplied in every depot at each period, the sure certain of bookings associated with each storage area at each period, a significant sample of bookings and returning containers that may become known in the near future and the expected transit times. Then decisions concerning the first time-period are implemented and, when in the following period, new information becomes available, data are adjusted, the problem is solved for the new current time and so on.

Moving the focus onto notational aspects, this optimization model considers a finite set $D$ of inland depots, a set $P$ of container types and a set $T$ of contiguous time-periods. Moreover, a subset $P_p$ of $P$ is defined, to represent the set of container types different from $p \in P$, that can be used to meet bookings of $p$-type containers. As proposed by Crainic et al. (1993-b), each inland depot is represented by two nodes, $j_t^p$ and $j_t^{p'}$, where $t \in T$ and $p \in P$, linked by a direct arc. The first node is related to the number $s_p^p \geq 0$ of empty containers of type $p \in P$, that become available at time $t \in T$ in that depot. The second node is associated with the number $d_p^p \geq 0$ of empty containers of type $p \in P$, that must be provided by that depot at time $t \in T$ to exporters.

In order to avoid infeasible solutions, due to the difference between supply and demand over the planning horizon, a port is considered and modelled through two nodes. The first one, denoted for each container type $p \in P$ by $h_p$, is associated with a demand $s_p$ of empty containers, that is equal to the regional surplus of empties. The second one, denoted for each container type $p \in P$ by $h'_p$, supplies a number $d_p$ of empty containers equal to the regional deficit of such equipment type. The next step consists of connecting each node $j$ to $h_p$ and $h'_p$ to each node $j'$.

The problem is presented as an integer programming model, whose decision variables are denoted by letter $x$ and costs by letter $c$:

- Variable $x(j_t^p, k_{t+\tau}^p)$ denotes the number flow of empty containers of type $p \in P$ moved from node $j \in D$ at time $t \in T$ to reach node $k \in D$ at time-period $t+\tau \in T$ ($\tau$ is the transit time between $j$ and $k$); $c(j_t^p, k_{t+\tau}^p)$ represents the related unitary cost.
New Optimization Models for Empty Container Management

- Variable $x(j^p_t, j'^p_t)$ denotes the number of empty containers of type $p \in P$ assigned at time $t \in T$ in such a depot to serve exporters; $c(j^p_t, j'^p_t)$ represents the related cost.

- Variable $x(j^p_t, j^p_{t+1})$ denotes the number of empty containers of type $p \in P$ stored in such a depot between time $t \in T$ and $t+1 \in T$; $c(j^p_t, j^p_{t+1})$ represents the related storage cost.

- Variable $x(j'_t, j'^p_t)$ denotes the number of empty containers of type $r \in P_p$, assigned at time $t \in T$ in such a depot to serve exporters, that ordered $p$-type containers; $c(j'_t, j'^p_t)$ represents the related cost. Although substitution costs depend on the origin-destination-pair, this study assumes a specific value associated with each depot and container-type.

- Variable $x(j^p_t, h^p_t)$ denotes the number of empty containers of type $p \in P$ moved from node $j \in D$ at time $t \in T$ to the port; $c(j^p_t, h^p_t)$ represents the related transportation cost.

- Variable $x(h^p_t, j'^p_t)$ denotes the number of empty containers of type $p \in P$ moved from the port to reach the node $j' \in D$ at time $t \in T$; $c(h^p_t, j'^p_t)$ represents the related transportation cost.

Since this study considers a heterogeneous fleet of empty containers having different size, the available storage space $U(j^p_t, j^p_{t+1})$ among times $t \in T$ and $t+1 \in T$ is expressed in terms of container numbers of a the largest type $\overline{p} \in P$. Then each type $r \neq \overline{p}$ is converted to type $\overline{p}$ using substitution factors $a_{rp}$ introduced by Crainic, et al. (1993-b). Moreover, to take into account balancing movements and agreements stipulated by carriers, the model considers a lower bound $l(j^p_t, k^p_{t+\tau})$ and an upper bound $u(j^p_t, k^p_{t+\tau})$ concerning the numbers of $p$-type empty containers moved from node $j \in D$ at time $t \in T$ to node $k \in D$ at time-period $t+\tau \in T$.

Figure 6-1 shows a sample network composed of three depots, denoted with letters $a$, $b$ and $c$, five time-periods and two container types, denoted with letters $p$ and $r$. While $p$-type containers can be used to serve bookings of $r$-type containers, the reverse substitution is not considered.
The proposed optimization model can be expressed as follows:
New Optimization Models for Empty Container Management

\[
\begin{align*}
\min & \sum_{i \in T} \sum_{j \in D} \left\{ \sum_{k \in D[k \neq j]} \left( \sum_{l \in D[l \neq j]} \left( c(j_i^p, k_{i+\tau}) \cdot x(j_i^p, k_{l+\tau}) + c(j_i^p, j_i^p) \cdot x(j_i^p, j_i^p) + c(j_i^p, j_{i+\tau}^p) \cdot x(j_i^p, j_{i+\tau}^p) + 
\right. \right. \\
& \left. \left. + \sum_{r \in D[r \neq j]} c(j_i^p, j_r^p) \cdot x(j_i^p, j_r^p) + c(j_i^p, h_{i+\tau}) \cdot x(j_i^p, h_{i+\tau}) \right) \right\} + \sum_{j \in D} c(h_{i+\tau}, j_i^p) \cdot x(h_{i+\tau}, j_i^p) \right\} \\
\end{align*}
\]

subject to

\[
\sum_{k \in D} \left[ x(j_i^p, k_{i+\tau}) - x(k_{i+\tau}, j_i^p) \right] + x(j_i^p, j_i^p) + x(j_i^p, j_{i+\tau}^p) - x(j_{i+\tau}^p, j_i^p) + \sum_{r \in D[r \neq j]} x(j_i^p, j_r^p) + \\
+x(j_i^p, h_{i+\tau}) = s_i^p \quad \forall j \in D, \forall t \in T, \forall p \in P \tag{6.2}
\]

\[
x(j_i^p, j_i^p) + \sum_{r \in D[r \neq j]} a_{ij} \cdot x(j_i^p, j_r^p) + x(h_{i+\tau}, j_i^p) = d_i^p \quad \forall j', \forall t \in T, \forall p \in P \tag{6.3}
\]

\[
\sum_{i \in T} \sum_{j \in D} x(j_i^p, h_{i+\tau}) = s_i^p \quad \forall p \in P \tag{6.4}
\]

\[
\sum_{i \in T} \sum_{j' \in D} x(h_{i+\tau}, j_i^p) = d_i^p \quad \forall p \in P \tag{6.5}
\]

\[
l(j_i^p, k_{i+\tau}) \leq x(j_i^p, k_{i+\tau}) \leq u(j_i^p, k_{i+\tau}) \quad \forall j \in D, \forall t \in T, \forall p \in P \tag{6.6}
\]

\[
x(j_i^p, j_{i+1}^p) + \sum_{r \in D[r \neq j]} a_{ij} \cdot x(j_i^p, j_{i+1}^p) \leq U(j_i^p, j_{i+1}^p) \quad \forall j \in D, \forall t \in T \tag{6.7}
\]

where \(t-\tau, t+\tau, t-1\) and \(t+1\) must belong to \(T\). All decision variables can have only non-negative integer values.

The objective function (6.1) represents the total cost of empty container management, including cost of transportation, substitution and storage in depots. Constraint set (6.2) is the flow conservation of \(p\)-type containers in node \(j \in D\) at time \(t \in T\). Constraint set (6.3) is the mass balance regarding containers of type \(p \in P\) in node \(j' \in D\) at time \(t \in T\). Constraint sets (6.4) and (6.5) ensure flow conservation in each nodes \(h_{i+\tau}\) and \(h_{i+\tau}^p\) for
each container type $p \in P$. Constraint set (6.6) guarantees the balancing movements among each pair of inland depots. Constraint set (6.7) ensures that the number of empty containers stored in each depot does not overcome the available storage space, expressed in number of containers of the specific type $\bar{p} \in P$. It is worth noting that such a constraint takes into account the different sizes of container types that use in a different way the storage capacity (for instance a 40 ft container is twice a 20 ft).

6.4 Some experimental results

Some computational tests are performed in order to verify the convergence rates of a standard algorithm for mixed integer programming. To solve numerical instances, written in MPS (Mathematical Programming Standard) format, the well-known solver Lindo 6.01 is adopted running on a 256MB 2.0GHz Pentium IV computer. Two container types are taken into account: the 40’ general purpose and 40’ high cube. According to standard logistic practices, the high cube container can replace the general purpose because it provides a larger volume. Due to the same reasons, the reverse substitution is not possible. Storage capacity is expressed in terms of 40’ft general purpose. Adopted conversion factors are shown in Table 6-1.

<table>
<thead>
<tr>
<th>Container type</th>
<th>$a_{rp}$</th>
<th>$a_{\bar{p}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 ft general purpose</td>
<td>$a_{rp} = 1$</td>
<td>$a_{\bar{p}} = 1$</td>
</tr>
<tr>
<td>40 ft high cube</td>
<td>$a_{rp} = 1$</td>
<td>$a_{\bar{p}} = 1$</td>
</tr>
</tbody>
</table>

Table 6-1. Conversion factors.

At first instances are solved without substitution options, balancing the network associated with each container type by the regional surplus/deficit (case 1). Such artificial numbers of containers are provided/requested by the port. As a result, the number of empty containers demanded and supplied is equal for each container type, all transportation requests can be met and feasible solutions are always obtained.

Then substitution variables are introduced (case 2), but a 1% tolerance factor has to be set to solve instances within reasonable time. Finally, better results are obtained by removing constraint sets (6.4) and (6.5) (case 3). Flow conservation constraints (6.2), (6.3) and non-negativity constraints allow performing a more suitable number of substitutions than in case 2. A 1% tolerance factor needs to be set in case 3 as well.

Table 6-2 shows the list of numerical tests performed.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Number of depots</th>
<th>Balancing Constraints</th>
<th>Substitution between container types</th>
<th>Number of variables</th>
<th>Objective function [€]</th>
<th>Tolerance</th>
<th>Iteration Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>7</td>
<td>case 1: Not allowed</td>
<td>1464</td>
<td>4273200</td>
<td>NO</td>
<td>313</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 2: Allowed</td>
<td>1660</td>
<td>3410340</td>
<td>1%</td>
<td>2073</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 3: Allowed</td>
<td>3410100</td>
<td></td>
<td></td>
<td>2953</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>14</td>
<td>case 1: Not allowed</td>
<td>3528</td>
<td>10149600</td>
<td>NO</td>
<td>634</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 2: Allowed</td>
<td>3948</td>
<td>8489410</td>
<td>1%</td>
<td>2291</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 3: Allowed</td>
<td>8286220</td>
<td></td>
<td></td>
<td>7764</td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>20</td>
<td>case 1: Not allowed</td>
<td>6841</td>
<td>20343800</td>
<td>NO</td>
<td>972</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 2: Allowed</td>
<td>7441</td>
<td>19545100</td>
<td>1%</td>
<td>4067</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 3: Allowed</td>
<td>16098900</td>
<td></td>
<td></td>
<td>8806</td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td>26</td>
<td>case 1: Not allowed</td>
<td>8955</td>
<td>24215700</td>
<td>NO</td>
<td>1245</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 2: Allowed</td>
<td>9741</td>
<td>23734400</td>
<td>1%</td>
<td>5432</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>case 3: Allowed</td>
<td>19863600</td>
<td></td>
<td></td>
<td>49651</td>
<td></td>
</tr>
</tbody>
</table>

Table 6-2. Instances solved by Lindo 6.01.

According to our tests, when substitutions are not taken into account, Lindo easily converges to the optimal solution. When substitution options are considered, better solutions can be achieved in terms of objective function. However, the problem becomes more computationally difficult to solve and a tolerance factor has to be set. Of course, such a factor can be reduced to reach optimal or near optimal solutions, but heavier computational efforts should be paid.

Moreover, it is worth noting that better solutions are obtained when constraints sets (6.4) and (6.5) are omitted. Their lack allows performing a more appropriate number of substitutions. Furthermore, constraints (6.2) and (6.3) ensure that, when regional surpluses take place, a suitable number of containers is provided by the port. They also guarantee that, when regional surpluses occur, unnecessary empty containers are moved to the port.

Finally, in order to reduce the gap among this model and real problems faced by local agencies, an important improvement will be made taking into account several ports. Such a development is proposed in the optimization model presented in the next paragraph.
6.5 An improved optimization model

This section describes an improved mathematical model taking into account decisions regarding rented containers. This model refers to the same network presented in paragraph 6.3 to manage company-owned containers. New sets of variables and constraints are introduced for this issue. The major advantage provided by rented containers consists of the opportunity to be reused or off-hired by local agencies according to their needs, taking into account agreements with lessors and some general rules imposed by the Head Office.

It is worth noting that contracts stipulated by lessors and shipping companies specify a per-diem rate and the maximum number of containers that can be picked-up and off-hired in certain depots every month. Moreover, drop-off fees are paid by shipping companies when containers are returned to lessors. Such fees are particularly expensive in surplus regions, where low business opportunities make the large part of empty containers useless for both lessors and shipping companies. Pick-up fees are applied when containers are rented in deficit areas, where many transportation opportunities take place.

Nowadays shipping companies are willing for logistic solutions to manage rented containers properly. Typically they adopt general rules depending on their business volume. During peak seasons the Head Office allows renting containers, whereas, during slack periods, local agencies are forced to reduce the fleet, so rented containers are returned to their owners. However, few tools have been proposed to validate the quality of this strategy.

The optimization model proposed in this paragraph assumes as decision variables the number of company-owned and rented containers to be moved among the different storage areas, the equipment to be stored in depots, substitutions among different types and the number of rented containers to be picked-up and dropped-off. This mathematical model aims to minimize the operational costs in a deterministic and dynamic environment, where decisions are implemented in a rolling horizon fashion. For sake of simplicity, a single lessor is considered in the proposed formulation. Its containers can be on-hired and off-hired in any depot of the region, paying the pick-up and drop-off fees specified in rental contacts.
Moving the focus onto notational aspects, this optimization model considers a finite set \( D \) of inland depots, a set \( H \) of ports, a set \( P \) of container types and a set \( T \) of contiguous time-periods. Moreover, a subset \( P_p \) of \( P \) is defined, to represent the set of container types different from \( p \in P \), that can be used to meet the requests of \( p \)-type containers. Each inland depot is represented by three nodes denoted by \( j_p^t \), \( j_p^t' \) and \( j_p^t'' \), where \( t \in T \) and \( p \in P \) (see Figure 6-2). The first node \( j_p^t \) is associated with the number \( s_p^t \geq 0 \) of containers-owned of type \( p \in P \), that become available at time \( t \in T \) in that depot. The second node \( j_p^t' \) is associated with the number \( d_p^t \geq 0 \) of empty containers of type \( p \in P \), that the depot must provide at time \( t \in T \) to exporters. The third one \( j_p^t'' \) is associated with the number \( s_p^t \geq 0 \) of rented-containers of type \( p \in P \), that become available at time \( t \in T \) in that depot. They can be assigned to bookings, hauled, stored or off-hired.

Requests of \( p \)-type containers can be satisfied both by owned and rented empties, so proper links from \( j_p^t \) to \( j_p^t' \) and from \( j_p^t'' \) to \( j_p^t' \) are considered. Since the number of containers available and requested in a given depot is normally not the same, several links are introduced among such nodes to allow inter-depot balancing transportation, substitutions and storage options.

According to rental contracts and other specific conditions like the peak or slack season, it is necessary to take into account the maximum numbers \( S_p \) and \( R_p \) of \( p \)-type containers that can be respectively on-hired and off-hired in a given month. This study takes into account a single lessor, that is modelled through a pair of artificial nodes \( \alpha_p \) and \( \beta_p \). The first one is associated with the maximum supply of \( S_p \) containers, whereas the other is related to the maximum demand of \( R_p \) containers. Then \( \alpha_p \) is linked to \( j_p^t'' \) and \( j_p^t \) to \( \beta_p \), \( \forall j'' \in D, \forall t \in T, \forall p \in P \).

Due to the difference among the number of containers supplied and required in that region, a set of ports is considered and modelled through two nodes. The first one, denoted by \( h^p \) for each port and each container type \( p \in P \), can provide a suitable number of containers. The second one, denoted by \( h^p \) for each port and each container type \( p \in P \), can provide a suitable number of containers.
type \( p \in P \), can receive a suitable number of containers. Then next step consists of connecting \( j_i^p \) to \( h^p \), \( \forall h \in H, \forall p \in P \) and \( h^p \) to \( j_i^p \), \( \forall h' \in H, \forall p \in P \).

Since this study considers a heterogeneous fleet of empty containers having different sizes, the available storage space at time is expressed in terms of numbers \( U(j_i^p, j_{i+1}^p) \) of containers of the largest type \( p \in P \). Then each type \( r \neq p \) is converted to type \( p \) using substitution factors \( a_{p,r} \) introduced by Crainic, et al. (1993-b). Moreover, to take into account balancing movements and agreements stipulated with carriers, this model considers a lower bound \( l(j_i^p, k_{i+1}^p) \) and an upper bound \( u(j_i^p, k_{i+1}^p) \) on the numbers of \( p \)-type empty containers moved from node \( j \in D \) at time \( t \in T \) to node \( k \in D \) at time-period \( t+\tau \in T \).

The problem is presented as an integer programming model, whose decision variables are denoted by letter \( x \) and costs by letter \( c \).

- **Variable** \( x(j_i^p, k_{i+1}^p) \) denotes the number of empty containers-owned of type \( p \in P \) hauled from node \( j \in D \) at time \( t \in T \) to reach node \( k \in D \) at time-period \( t+\tau \in T \) (the transit time \( \tau \) between \( j \) and \( k \) may be lower than the time period); \( c(j_i^p, k_{i+1}^p) \) represents the related unitary cost.

- **Variable** \( x(j_i^p, j_{i+1}^p) \) denotes the number of empty containers-owned of type \( p \in P \) allocated at time \( t \in T \) in such a depot to serve exporters; \( c(j_i^p, j_{i+1}^p) \) represents the related cost.

- **Variable** \( x(j_i^p, j_{i+1}^p) \) denotes the number of empty containers-owned of type \( p \in P \) stored in such a depot between time \( t \in T \) and \( t+1 \in T \); \( c(j_i^p, j_{i+1}^p) \) represents the related storage cost.

- **Variable** \( x(j_i^p, j_{i+1}^p) \) denotes the number of empty containers-owned of type \( r \in P_p \), allocated at time \( t \in T \) in such a depot to serve exporters, that ordered \( p \)-type containers; \( c(j_i^p, j_{i+1}^p) \) represents the related cost.

- **Variable** \( x(j_i^p, h^p) \) denotes the number of empty containers-owned of type \( p \in P \) moved from node \( j \in D \) at time \( t \in T \) to the port \( h \in H \); \( c(j_i^p, h^p) \) represents the related transportation cost.
• Variable \( x(h^p, j^p_t) \) denotes the number of empty containers-owned of type \( p \in P \) moved from the port \( h \in H \) to reach the node \( j' \in D \) at time \( t \in T \); \( c(h^p, j^p_t) \) represents the related transportation cost.

• Variable \( x(j''^p, k''^p_{t+\tau}) \) denotes the flow of rented empty containers of type \( p \in P \) moved from node \( j'' \in D \) at time \( t \in T \), reaching node \( k'' \in D \) at time-period \( t+\tau \in T \) (\( \tau \) is the transit time among such nodes); \( c(j''^p, k''^p_{t+\tau}) \) represents the related unitary cost. Compared to transportation costs of company-owned containers, such costs should be properly set to consider the per-diem daily rental fee.

• Variable \( x(j'^p_t, j''^p_t) \) denotes the number of rented empty containers of type \( p \in P \) allocated at time \( t \in T \) in such a depot to serve exporters; \( c(j'^p_t, j''^p_t) \) represents the related unitary cost. Compared to corresponding costs of company-owned containers, such costs should be properly set to take into account the per-diem rental fee.

• Variable \( x(j''^p_t, j'^p_{t+1}) \) denotes the number of rented empty containers of type \( p \in P \) stored in such a depot between time \( t \in T \) and \( t+1 \in T \); \( c(j''^p_t, j'^p_{t+1}) \) represents the related unitary cost. Compared to transportation costs of company-owned containers, such costs should be properly set to take into account the per-diem rental fee.

• Variable \( x(j''^r_t, j'^p_t) \) denotes the number of rented empty containers of type \( r \in P_p \) allocated at time \( t \in T \) in such a depot to serve exporters, that ordered \( p \)-type containers; \( c(j''^r_t, j'^p_t) \) represents the related unitary cost. Compared to transportation costs of company-owned containers, such costs should be properly set to take into account the per-diem daily fee.

• Variable \( x(j''^p_t, \beta^p) \) denotes the number of rented empty containers of type \( p \in P \) returned to their owners at time \( t \in T \) in the terminal; \( c(j''^p_t, \beta^p) \) represents the related drop-off cost. To take into account the monthly number of containers that can be off-hired in each depot, this variable is also characterized by the upper bound \( u(j''^p_t, \beta^p) \).
Variable \( x(\alpha^p, j''^p) \) denotes the number of rented empty containers of type \( p \in P \) passed under the control of the shipping company at time \( t \in T \) in the terminal through the artificial node \( \alpha_p \); \( c(\alpha^p, j''^p) \) represents the related pick-up cost. To take into account the monthly number of containers that can be picked-up in each depot, this variable is also characterized by the upper bound \( u(\alpha^p, j''^p) \).

Figure 6-2 shows a sample network composed of three depots, denoted by letters \( a, b \) and \( c \), one port, five time-periods and one container type, denoted by letter \( p \). The proposed optimization model can be expressed as follows:

\[
\begin{align*}
\text{min} & \sum_{t \in T} \sum_{p \in P} \left[ \sum_{j \in D} \left( \sum_{j'' \in D} c(j''^p, k''^p) \cdot x(j''^p, k''^p) + c(j''^p, j''^p) \cdot x(j''^p, j''^p) + c(j''^p, j''^p) \cdot x(j''^p, j''^p) \right) \right] + \sum_{r \in P, j \in D} \left( \sum_{j'' \in D} c(j''^p, k''^p) \cdot x(j''^p, k''^p) + c(j''^p, j''^p) \cdot x(j''^p, j''^p) \right) + \\
& + \sum_{j'' \in D} c(j''^p, k''^p) \cdot x(j''^p, k''^p) + \sum_{j \in D} x(j''^p, j''^p) \cdot x(j''^p, j''^p) \right] + \sum_{r \in P, j \in D} \left( \sum_{j'' \in D} c(j''^p, k''^p) \cdot x(j''^p, k''^p) + c(j''^p, j''^p) \cdot x(j''^p, j''^p) \right) + \\
& + \sum_{j'' \in D} c(j''^p, k''^p) \cdot x(j''^p, k''^p) \right] + \sum_{r \in P, j \in D} \left( \sum_{j'' \in D} c(j''^p, k''^p) \cdot x(j''^p, k''^p) + c(j''^p, j''^p) \cdot x(j''^p, j''^p) \right) \right] \\
\text{subject to} & \sum_{k \in D} \left[ x(j''^p, j''^p) - x(k''^p, j''^p) \right] + x(j''^p, j''^p) + x(j''^p, j''^p) - x(j''^p, j''^p) + \sum_{r \in P, j \in D} x(j''^p, j''^p) + \\
& + \sum_{h \in H} x(j''^p, j''^p) = s_j^{\alpha_p} \quad \forall j \in D, \forall t \in T, \forall p \in P \quad (6.11) \\
& x(j''^p, j''^p) + \sum_{r \in P, j \in D} a_{p_r} \cdot x(j''^p, j''^p) + \sum_{h \in H} x(h''^p, j''^p) + x(j''^p, j''^p) + \sum_{r \in P, j \in D} a_{p_r} \cdot x(j''^p, j''^p) = d_j^{\alpha_p} \\
& \quad \forall j'' \in D, \forall t \in T, \forall p \in P \quad (6.12) \\
& \sum_{k \in D} \left[ x(j''^p, j''^p) - x(k''^p, j''^p) \right] + x(j''^p, j''^p) - x(j''^p, j''^p) + x(j''^p, j''^p) + \\
& + \sum_{r \in P, j \in D} x(j''^p, j''^p) + x(j''^p, j''^p) - x(\alpha^p, j''^p) = s_j^{\alpha_p} \quad \forall j'' \in D, \forall t \in T, \forall p \in P \quad (6.13)
\end{align*}
\]
\[ \sum_{t \in T} \sum_{j \in D \cup H} x(\alpha_p, j''^p) \leq S^p \quad \forall p \in P \]  

\[ \sum_{t \in T} \sum_{j \in D \cup H} x(j''^p, \beta^p) \leq R^p \quad \forall p \in P \]  

\[ x(j''^p, \beta^p) \leq u(j''^p, \beta^p) \quad \forall j'' \in D, \forall t \in T, \forall p \in P \]  

\[ x(\alpha_p, j''^p) \leq u(\alpha_p, j''^p) \quad \forall j'' \in D, \forall t \in T, \forall p \in P \]  

\[ l(j_i, k^p_{i+1}) \leq x(j_i, k^p_{i+1}) + x(j''_{i+1}, k''_{i+1}) \leq u(j_i, k_{i+1}) \quad \forall j_i \in D, \forall t \in T \]  

\[ x(j_i, j_{i+1}) + x(j''_{i}, j''_{i+1}) + \sum_{r \in P} a_r \cdot x(j_r, j_{i+1}) + \sum_{r \in P} a_r \cdot x(j''_r, j''_{i+1}) \leq U(j_i, j_{i+1}) \quad \forall j \in D, \forall t \in T \]  

where \( t-\tau, t+\tau, t-I \) and \( t+I \) must belong to \( T \). All decision variables can have only non-negative integer values.

The objective function (6.10) represents the total cost of empty container management, including cost of transportation, substitution, storage, pick-up and drop-off. Constraint set (6.11) is the flow conservation of \( p \)-type containers in node \( j_i \), \( \forall j \in D, \forall t \in T, \forall p \in P \). It requires to use the available company-owned containers to serve exports.

Constraint set (6.12) is the mass balance regarding containers of type \( p \in P \) in node \( j_i \), \( \forall j \in D, \forall t \in T, \forall p \in P \). It requires to satisfy all bookings using both company-owned and rented containers. Constraint set (6.13) is the flow conservation of \( p \)-type containers in node \( j'' \), \( \forall j'' \in D, \forall t \in T, \forall p \in P \). Moreover constraints (6.11) and (6.12) guarantee that, when regional surpluses take place, a suitable number of containers is provided by ports. They also ensure that, when regional surpluses occur, unnecessary empty containers are moved to ports.
New Optimization Models for Empty Container Management

Figure 6-2. A sample dynamic network with both company-owned and rented containers.
Constraint set (6.14) and (6.15) require that the local agency can on-hire and off-hire at most the maximum number of empty containers specified in rental contracts. Constraint set (6.16) and (6.17) ensure that the local agency does not off-hire and on-hire in each depot a number of empty containers larger than the value indicated in rental contracts. Constraint set (6.18) ensures the balancing movements among each pair of inland depots for both company-owned and rented containers. Constraint set (6.19) guarantees that the number of empty containers stored in each depot does not overcome the available storage space, expressed by the number of containers of the specific type \( \bar{p} \in \mathcal{P} \). It is worth noting that such a constraint takes into account the different sizes of container types that use the storage capacity in a different way.

6.6 Case study

Some computational tests are performed in order to verify the convergence rates of some standard algorithms for mixed integer programming. To solve numerical standard 15-periods instances, the commercial solver Cplex-mipopt 7.5 (CPLEX optimization, 1995) is used running on a 256MB 1.7 GHz Pentium IV computer. Eight container types are taken into account. Since storage capacity is expressed in terms of 40’ft general purpose, Table 6-3 shows the adopted conversion factors.

<table>
<thead>
<tr>
<th>Container type</th>
<th>( a_{rp} )</th>
<th>( a_{\bar{p}} )</th>
<th>Substitutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 ft high cube</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>40 ft general purpose</td>
<td>1</td>
<td>1</td>
<td>It can be replaced by a 40 ft high cube container</td>
</tr>
<tr>
<td>20 ft general purpose for heavy goods</td>
<td>1</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>20 ft general purpose</td>
<td>1</td>
<td>0.5</td>
<td>It can be replaced by a 20 ft general purpose container for heavy goods</td>
</tr>
<tr>
<td>40 ft flat</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>40 ft open top</td>
<td>1</td>
<td>1</td>
<td>It can be replaced by a 40 ft flat container</td>
</tr>
<tr>
<td>20 ft flat</td>
<td>1</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>20 ft open top</td>
<td>1</td>
<td>0.5</td>
<td>It can be replaced by a 20 ft flat container</td>
</tr>
</tbody>
</table>

Table 6-3. Container types, conversion factors and substitutions.
Hereafter table 6-4 presents the lists of tests performed.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Number of Depots</th>
<th>Number of Types</th>
<th>Number of constraints</th>
<th>Number of Variables</th>
<th>Time for Cplex (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>60</td>
<td>2</td>
<td>6663</td>
<td>18766</td>
<td>1.03</td>
</tr>
<tr>
<td>P2</td>
<td>60</td>
<td>4</td>
<td>11985</td>
<td>37525</td>
<td>3.52</td>
</tr>
<tr>
<td>P3</td>
<td>60</td>
<td>6</td>
<td>17245</td>
<td>56291</td>
<td>5.45</td>
</tr>
<tr>
<td>P4</td>
<td>60</td>
<td>8</td>
<td>22529</td>
<td>75057</td>
<td>10.24</td>
</tr>
</tbody>
</table>

Table 6-4. Instances solved by Cplex 7.5.

The commercial solver Cplex has shown excellent performance in terms of user waiting time. Such a characteristic represents a major requirement for local agencies, that need to be reactive in the complex industry of container-based transportation.

6.7 Conclusions and future developments

Optimization models presented in this chapter address a wide array of decisions for the local management of empty containers. Starting from the seminal framework by Crainic et al., (1993-b), new specific constraints are introduced to take into account the available storage space for a heterogeneous fleet of empty containers. Both contributions have determined specific decisions to be implemented by inland depots. Moreover, we have identified the number of empty containers dispatched to/from ports to fulfill customer needs. Such flows represent supply/demand parameters for ports in the maritime reposition issue (see Chapter 7 for details).

In the first model we have concluded that substitution strategies can significantly improve the obtained results. However, sometimes a tolerance factor must be set to reach optimal or near optimal solutions, depending on the adopted solver. In the second model we have planned when and where containers should be on-hired and off-hired, taking into account the main clauses of rental contracts. Although huge in variables and constraints, this model has been efficiently solved by Cplex.

From an algorithmic point of view, both mathematical models exhibits strong particular algebraic structures, that can be used to exploit and to develop effective and specialized algorithms. For example, if it were not for substitution arcs, the model could be converted into a multicommodity problem, for which some specific and efficient algorithms already exist. In this context, further research is in progress making recourse
to decomposition techniques based on lagrangian relaxation and bundle or subgradient algorithms (Bertsekas, 2003).

Moreover, since the parameters of the problem are assumed to be known in future periods, the proposed formulation is deterministic. As a future research, scenario analysis formulation is in progress to provide a more detailed representation of the sources of uncertainty involved in this issue.
Chapter 7

An optimization model for empty container maritime reposition under uncertainty

7.1 Introduction

A significant factor for the competitiveness of shipping companies is the availability of empty containers in ports to support the activity of their local agencies. Due to the global trade imbalance, some ports tend to accumulate empty containers, resulting in unnecessary storage costs. Whereas others face shortages that expose local agencies to the risk of competitors providing containers as requested. As a consequence, shipping companies must be reactive to meet their needs and perform the maritime reposition of their empty containers.

Although shipping companies have a maritime network to reallocate empties, repositioning is not a straightforward task. Several sources of uncertainty occur, like the number of containers requested in the future, the time when empties will come back to ports, and transportation capacities. As explained in Chapter 4, the shipping industry will significantly benefit from the development of a tool which is able to determine where and when empty containers should be repositioned.

The scientific literature in the field has proposed both deterministic (Choong et al., 2002; Olivo et al., 2005) and stochastic (Cheung and Chen, 1998) optimization models. On the one hand, deterministic models take into account a single realization of uncertain parameters over time. On the other hand, stochastic programming requires a good knowledge of random variable distributions to yield effective solutions. For example, when probabilities associated with extreme scenarios are underestimated, high-risk decisions may be produced. Moreover, local agencies are not used to build adequate statistics on the regional surplus/deficit for empty containers and the Head Office cannot plan their reposition using such parameters.

This study proposes a dynamic optimization model to solve the maritime reposition issue for a heterogeneous fleet of containers, taking into account uncertainty through a set of representative scenarios. This method takes into account a set $G$ of statistically independent scenarios, in which each scenario represents a potential realization of
uncertain parameters over time. A weight $w_k$ can be assigned to each scenario to characterize its relative importance. Weights may represent probabilities of occurrence or, when reliable probabilistic rules are unavailable, subjective parameters assigned by managers on the basis of their own skill. The resulting mathematical model is named chance-model to indicate that is not statistically based.

Since several scenarios are included in a single dynamic optimization model, large scale instances must be solved in reasonable time for the operating needs of the shipping industry. The commercial solver Cplex (1995) is used for this purpose. Such an optimization model exhibits some strong algebraic structures, sometime nested in different ways. They can be exploited to develop specialized resolution techniques based on decomposition algorithms, lagrangian relaxations and structured interior point methods (Castro, 2003).

This approach keeps validity in the context of the so-called deterministic equivalent formulation. It is based on the tree structure derived from the aggregation of parallel scenarios according to the “progressive hedging principle” (Rockafellar and Wets, 1991). To exploit the strong algebraic structures of the deterministic equivalent formulation, this study proposes a lagrangian relaxation of inter-stage constraints and performs a subsequent computation of lagrangian multipliers employing a freeware bundle algorithm.

The remainder of this chapter is as follows. In paragraph 7.2 we review the most significant contributions in the empty container reposition issue. Paragraph 7.3 illustrates the set of parameters know when decisions must be made. Paragraph 7.4 formalizes the optimization model. In paragraph 7.5, several instances are solved by Cplex and the most significant results are provided. Paragraph 7.6 introduces the scenario-tree formulation associated with the previous model, describes the resolution technique adopted and provides some results.

7.2 Literature review

In recent years some shipping companies have adopted decision supporting systems to reposition containers using mathematical programming models and algorithms. Such models are usually deterministic, resort to statistical bases to calculate uncertain parameters and provide decisions to be implemented in a rolling horizon fashion. The
current application of mathematical programming techniques to repositioning issues is the consequence of the intensive research developed in past years.

Dejax and Crainic (1987) reviewed past contributions regarding the management of flows. In their opinion, although this topic had received much attention since the sixties, little consideration had been dedicated to the development of specific models in the container transportation issue. They mentioned few authors addressing the problem of allocating empty containers in a deterministic and dynamic environment. Network optimization and linear programming algorithms were usually adopted.

Crainic, et al. (1989) discussed the problem of locating empty containers in an intercity freight transportation system on behalf of a shipping company. They proposed an optimization model in order to minimize the cost of depot opening and container transportation, while satisfying customer demands. This issue belongs to a strategic planning level and identifies a suitable set of inland depots supporting the activity of ports.

Crainic, et al. (1993-b) presented a general framework to address the specific characteristics of the empty container allocation problem in the context of a land distribution system for a maritime shipping company. They developed a deterministic dynamic single-commodity model and a multicommodity dynamic variance in order to minimize total inland operating costs. They also provided a mathematical formulation to deal with the stochastic characters of demands and supplies. No experimental result was provided.

Lai et al. (1995) investigated the dynamic and stochastic container management issue through simulation models. They tried to evaluate several allocation policies to prevent the shortage of empty containers when they are requested. However, the huge sizes of numerical problems resulted in a problematical testing phase.

Shen and Khoong (1995), focusing on the business perspective of the shipping industry, developed a decision supporting system for the maritime reposition of empty containers. They proposed a deterministic dynamic formulation based on a network optimization model to minimize transportation costs. Moreover, it can provide decisions regarding leasing activities. They also suggested network algorithms as resolution techniques, but no experimental test was provided.
Since in the real world it is necessary to deal with stochastic systems, Cheung and Chen (1998) applied the network recourse formulation to the dynamic empty container allocation problem in an international maritime system. Randomness arose from demands for and supplies of empty containers in ports and from vessel capacities for empty containers. Although shipping companies typically manage heterogeneous fleets of containers, the authors considered one container type. Moreover, while the authors considered some probabilistic rules to estimate uncertain parameters, reliable statistics are often unavailable.

Jiele (1999) investigated the maritime reposition issue by proposing two dynamic optimization models. He adopted a minimum cost flow algorithm for the single commodity case and a linear programming technique for the multicommodity variance. Some experimental results are provided to show the relation between problem size and computational times. His formulation is deterministic.

Choong et al. (2002) addressed the end-of-horizon issue in a land distribution system. They proposed a deterministic and dynamic optimization model taking into account a homogeneous fleet of empty containers. The authors concluded that the application of a longer planning horizon usually results in better distribution plans.

Leung and Wu (2004) developed a dynamic optimization model for the dynamic reposition of empty containers from surplus ports to demand ports. They considered a set of scenarios associated with different realizations of customer demands. Their solution is robust because it is insensitive to data noise and uncertain parameters. Our contribution emphasizes another aspect of robustness, in the sense that the risk of wrong decisions is minimized.

Olivo et al. (2005) proposed a time-extended optimization model to support the decisions of shipping companies in the context of a multimodal network. However, their formulation is still deterministic and uncertain parameters are taken into account by implementing decisions in a rolling horizon fashion. Moreover, their model did not capture some specific characteristics of the maritime reposition issue, like the restrictions imposed by different ports.

Erera et al. (2005) proposed a dynamic large-scale optimization model to manage loaded and empty tank containers simultaneously. The incorporation of routing and reposition decisions in a single multicommodity model resulted in a significant drop of
operating costs. However, their formulation is still deterministic and uncertain parameters are considered in a rolling horizon fashion.

Di Francesco et al. (2006) proposed a deterministic dynamic optimization model to address the inland allocation and the maritime reposition of empty containers. However, taking into account the logistic practices adopted by shipping companies, inland allocation and maritime reposition should be addressed by different optimization models.

7.3 Decisions variables and parameters

In order to model the maritime reposition issue properly, two classes of ports have to be considered. Ports of the first class have enough storage space and make decisions on empty containers after their arrival from the landside. In this case the Head Office must decide immediately about the known number of containers arriving from the landside and stored from previous periods. It must call for how many will be stored and how many will be assigned to vessels arriving after 24 hours. This interval of time has been suggested by some industrial experts to indicate the minimum dwelling time for outbound containers in ports.

The ports of the second class force shipping companies to indicate the vessels for empty containers before their arrival from the landside. In this context, local agencies inform the Head Office about the expected number of empty containers arriving in ports in the following 24 hours. In this case the Head Office must assign an uncertain number of containers to vessels, that will berth starting from 48 hours. The expected number of containers arriving at ports will become known the next day, when such ports will observe how many empties will have already arrived. Therefore, the Head Office must consider different scenarios regarding empty containers arriving from the landside, come to the potential decisions for each possible evolution and implement only decisions corresponding to the scenario that will have occurred.

Figure 7-1 shows a time-extended network made up of one port of the first class, denoted by letter B and two ports of the second one, denoted by letters A and C. In order to model the storage of unloaded containers, both ports A and C are split into two nodes. Numbers from 1 to 4 indicate four lines operated by four different vessels. For instance, vessel 1 will arrive in period 4 at port B. This port can load on this vessel.
empty containers available from the third period and unload empties, that can reach inland destinations starting from the fifth period.

Figure 7-1 also differentiates among past, current and future decisions. It is worth noting that the Head Office cannot decide immediately about outbound empty containers that have become available from the first period in port A and C, because it was forced to make such decisions before their arrival from the landside. In port B the Head Office can determine the most suitable destination for empty containers that have become available until the decision-time.

In order to satisfy the demand for empty containers, the Head Office must decide straightaway how many empties will be stored, unloaded from vessels arriving in the second period and how many will be kept on vessels. Since unexpected bookings can suddenly appear over time, this study assumes that the demand is known just for the first two periods of the planning horizon.

To make matters worse, some vessels may have uncertain composition when decisions must be made. To clarify, vessels travelling over short distances do not offer sure information, because they can be still berthed at the previous port, when the Head Office must determine loading/unloading operations in the next one. In Figure 7-1 the composition of vessel 3 is already known, while the others are uncertain. As a result, the
Head Office must decide now how many empty containers will be loaded and unloaded from vessels 1, 2 and 4 arriving in period two, even if their composition is unsure. Furthermore, at the decisional time the Head Office knows the list of loaded containers to be put on and unloaded from vessels arriving in the second period. Since loaded containers have a greater priority and unexpected bookings must be met, the residual capacity for empties on vessels is an uncertain parameter (Cheung and Chen, 1998). Since the demand of empty containers is known for the first two periods, decisions to be made now must be the same for every scenario up to the second period. However, as shown in Figure 7-1, the supply of empty containers in the second period in ports A and C is uncertain and the number of empty containers carried by vessel 1, 2 and 4 is not sure. This study assumes that such parameters are known because shipping companies can estimate their value. To clarify, local agencies know the number of empty containers to be shipped from the landside to the port and, if no failure or accident occurs, such containers will already reach ports. Moreover the planner of the Head Office can indicate the most probable composition of vessels before the conclusion of terminal operations.

The time interval between the decisions made by the Head Office and the arrival of vessels enables ports to plan their internal activity properly (for instance they have to organize the so-called housekeeping). Moreover such decisions will be exploited by the same shipping companies to negotiate properly the time-window for terminal operation and avoid cut-and-run problems.

7.4 Optimization model

In this paragraph we propose an optimization model to manage a heterogeneous fleets of empty containers over a maritime network. The Head Office must make decisions concerning the number of containers repositioned, stored, loaded, unloaded and kept on vessels. As detailed before, we assume an operational environment made up of two categories of ports, according to the different rules imposed on empty containers. We consider the day as time-step of a multi-period network, because this interval of time represents the minimum dwelling time of containers in ports. Moreover the shortest sailing time in maritime networks is usually longer than a day. We implement decision
in a rolling horizon fashion, i.e. decisions are implemented in an initial subset of the planning horizon.

Regarding notation, we consider a set $P$ of container types, a set $T$ of contiguous time-periods, a set $V$ of vessels, and a set $G$ of scenarios associated with weights $w_g, g \in G$. Let $H_1$ be the set of ports in which unassigned empty containers can be stored and let $H_2$ represent the set of ports in which this option is not allowed. Furthermore, we denote by $t'$ the interval of time between the arrival of empty containers at ports of set $H_2$ and the berthing of vessels, to which such containers have to be assigned.
The notation \( b_{i,t}^{p,g} \geq 0 \) represents for a port \( i \in H_1 \) the supply (demand) of empty containers of type \( p \in P \) at time \( t \in T \) in scenario \( g \in G \). Each port in \( H_2 \) is represented by two nodes \( i \) and \( i' \). The first node \( i \in H_2 \) is related to the supply \( s_{i,j}^{p,g} \) of empty containers of type \( p \in P \) available in that port at time \( t \in T \) in scenario \( g \in G \). The node \( i' \in H_2 \) is associated with the demand \( d_{i,j}^{p,g} \) of empty containers of type \( p \in P \) requested in that port at time \( t \in T \) in scenario \( g \in G \). Proper capacity constraints are proposed to avoid storing and repositioning an inadmissible number of empty containers. Moreover, since we are in dealing with the management of a heterogeneous fleet of containers of different sizes, we consider the largest container type \( \overline{p} \) and express capacities in terms of number of available slots able to include \( \overline{p} \)-type containers. Given the container type \( q \neq \overline{p} \), the available space for \( q \)-type containers can be determined using conversions factor \( a_{pq} \), introduced by Crainic, et al. (1993-b). In the notation adopted hereafter, \( U^h(i^p_t, i^p_{t+1}) \) represents the storage capacity of port \( i \in H_1 \), \( U^h(i^p_t, i^p_{t+1}) \) the storage capacity of port \( i \in H_2 \), and \( U^r(v_{g}, (i^p_t, j^p_{t+1}) \) the residual capacity for empty containers carried by vessel \( k \in V \) travelling among ports \( i \in H_1 \cup H_2 \) and \( j \in H_1 \cup H_2 \) in scenario \( g \in G \). The sources of uncertainty involved in the issue are \( b_{i,t}^{p,g}, s_{i,j}^{p,g}, d_{i,j}^{p,g}, \) and \( U^r(v_{g}, (i^p_t, j^p_{t+1}) \). The problem is presented as an integer programming model whose decision variables are denoted by letter \( x \) and costs by letter \( c \), where \( l \) means “loaded”, \( u \) “unloaded”, \( r \) “repositioned”, and \( h \) “hold”.

1. Variable \( x^l_g(i^p_t, v_{p,i+1}^p) \) indicates the number of empty containers of type \( p \in P \), available in port \( i \in H_1 \) at time \( t \in T \), to be loaded on vessel \( v \in V \) arriving at time \( (t+l) \in T \) in scenario \( g \in G \); \( c^l(i^p_t, v_{p,i+1}^p) \) represents the related unitary cost.

2. Variable \( x^u_g(i^p_t, v_{p,i+1}^p) \) indicates the number of empty containers of type \( p \in P \), available in port \( i \in H_2 \) at time \( t \in T \), to be loaded on vessel \( v \in V \) arriving at time \( (t+t') \in T \) in scenario \( g \in G \); \( c^l(i^p_t, v_{p,i+1}^p) \) represents the related unitary cost.

3. Variable \( x^h_g(v^p_t, i^p_{i+1}) \) indicates the number of empty containers of type \( p \in P \) to be unloaded in scenario \( g \in G \) from vessel \( v \in V \) arriving at time \( t \in T \) at port...
New Optimization Models for Empty Container Management

\[ i \in H_1, \text{ where they become available at time } (t+1) \in T; \quad c^u(v_i^p, i_{t+1}^p) \text{ represents the related unitary cost.} \]

4. Variable \( x_g^p(v_i^p, i_{t+1}^p) \) indicates the number of empty containers of type \( p \in P \), to be unloaded in scenario \( g \in G \) from vessel \( v \in V \) arriving at time \( t \in T \) at port \( i' \in H_2 \), where they become available at time \( (t+1) \in T \); \( c^u(v_i^p, i_{t+1}^p) \) represents the related unitary cost.

5. Variable \( x_{v,g}^p(i_t^p, j_{t+\tau}^p) \) indicates the number of empty containers of type \( p \in P \), to be repositioned in scenario \( g \in G \) by vessel \( v \in V \) between ports \( i \in H_1 \cup H_2 \) and \( j \in H_1 \cup H_2 \) with respective berthing time \( t \in T \) and \( (t+\tau) \in T \); \( c_r^u(i_t^p, j_{t+\tau}^p) \) represents the related unitary cost.

6. Variable \( x_i^p(i_t^p, i_{t+1}^p) \) indicates the number of not-yet-assigned empty containers of type \( p \in P \) to be stored in port \( i \in H_1 \) between times \( t \in T \) and \( (t+1) \in T \) in scenario \( g \in G \); \( c^h(i_t^p, i_{t+1}^p) \) represents the related unitary cost.

7. Variable \( x_g^p(i_t^p, i_{t+1}^p) \) indicates the number of empty containers of type \( p \in P \) to be stored in port \( i \in H_2 \) between times \( t \in T \) and \( (t+1) \in T \) in scenario \( g \in G \); \( c^h(i_t^p, i_{t+1}^p) \) represents the relative cost.

The resulting mathematical model can be expressed as follows:

\[
\begin{align*}
\min & \quad \sum_{g \in G} w_g \left[ \sum_{i \in H_1} \sum_{p \in P} \left[ \sum_{i_{t+1}^p \in I_{t+1}} c^h(i_t^p, i_{t+1}^p)x_g^p(i_t^p, i_{t+1}^p) + \sum_{v \in V} c^u(v_i^p, v_{t+1}^p)x_g^p(i_t^p, v_{t+1}^p) \right] + \\
& + \sum_{i \in H_2} \sum_{j_{t+\tau}^p \in J_{t+\tau}} c_r^h(i_t^p, j_{t+\tau}^p)x_{v,g}^p(i_t^p, j_{t+\tau}^p) + \sum_{i \in H_2} \sum_{v \in V} c_r^u(v_i^p, v_{t+\tau}^p)x_{v,g}^p(i_t^p, v_{t+\tau}^p) + \\
& + \sum_{v \in V} \left[ c_g^u(v_t^p, i_{t+1}^p)x_g^p(v_t^p, i_{t+1}^p) + \sum_{i \in H_1} c_g^u(v_t^p, i_{t+1}^p)x_g^p(v_t^p, i_{t+1}^p) + \sum_{v \in V} c_g^u(v_t^p, i_{t+1}^p)x_g^p(v_t^p, i_{t+1}^p) \right] \right] \\
\text{subject to } & \quad \sum_{v \in V} x_g^p(i_t^p, v_{t+1}^p) + x_g^h(i_t^p, i_{t+1}^p) - \sum_{v \in V} x_g^u(v_{t+1}^p, i_t^p) - x_g^h(i_{t+1}^p, i_t^p) = b_g^{p,s} \\
& \forall i \in H_1, \forall t \in T, \forall p \in P, \forall g \in G
\end{align*}
\]
New Optimization Models for Empty Container Management

\[ \sum_{v \in F} x_g(i^p_r, v_{i+r}) = s^{p,g}_{i,t} \quad \forall i \in H_2, \forall t \in T, \forall p \in P, \forall g \in G \quad (7.3) \]

\[ x^h_g(i^p_r, i^p_{i+r}) - \sum_{v \in F} x^u_g(v_{i-1}, i^p_r) - x^h_g(i^p_{i-1}, i^p_r) = d^{p,g}_{i,t} \quad \forall i' \in H_2, \forall t \in T, \forall p \in P, \forall g \in G \quad (7.4) \]

\[ x^r_{v,g}(i^p_r, j^p_{i+t}) + x^u_g(v_{i}^p, i^p_r) + x^u_g(v_{i}^p, i^p_{i+t}) - x^r_{v,g}(j^p_{i-1}, i^p_r) - x^l_g(i^p_{i-1}, v_{i}^p) - x^l_g(i^p_{i-1}, v_{i}^p) = 0 \quad \forall v \in V, \forall t \in T, \forall p \in P, \forall g \in G \quad (7.5) \]

\[ x^h_g(i^p_r, i^p_{i+t}) + \sum_{q \in P \mid q \neq p} a_{p,q} \cdot x^h_g(i^q_r, i^q_{i+t}) + \sum_{v \in F} x^l_g(i^p_r, v_{i+t}) + \sum_{q \in P \mid q \neq p} \sum_{v \in F} x^l_g(i^q_r, v_{i+t}) \leq U^h(i^p_r, i^p_{i+t}) \quad \forall i \in H_1, \forall t \in T, \forall g \in G \quad (7.6) \]

\[ x^h_g(i^p_r, i^p_{i+t}) + \sum_{q \in P \mid q \neq p} a_{p,q} \cdot x^h_g(i^q_r, i^q_{i+t}) + \sum_{v \in F} x^l_g(i^p_r, v_{i+t}) + \sum_{q \in P \mid q \neq p} \sum_{v \in F} x^l_g(i^q_r, v_{i+t}) \leq U^h(i^p_r, i^p_{i+t}) \quad \forall i' \in H_2, \forall t \in T, \forall g \in G \quad (7.7) \]

\[ x^r_{v,g}(i^p_r, j^p_{i+t}) + \sum_{q \in P \mid q \neq p} a_{p,q} \cdot x^r_{v,g}(i^q_r, j^q_{i+t}) \leq U^r_{v,g}(i^p_r, j^p_{i+t}) \quad \forall v \in V, \forall t \in T, \forall g \in G \quad (7.8) \]

\[ x^l_g(i^p_r, v_{i+t}) = x^l_g(i^p_r, v_{i+t}) \quad \forall i \in H_1, t = \{1,2\}, \forall p \in P, \forall g, f \in G \quad (7.9) \]

\[ x^l_g(i^p_r, v_{i+t}) = x^l_g(i^p_r, v_{i+t}) \quad \forall i \in H_2, t = \{1,2\}, \forall p \in P, \forall g, f \in G \quad (7.10) \]

\[ x^u_g(v_{i+t}, j^p_{i+t}) = x^u_g(v_{i+t}, j^p_{i+t}) \quad \forall i \in H_1, t = \{1,2\}, \forall p \in P, \forall g, f \in G \quad (7.11) \]

\[ x^u_g(v_{i+t}, j^p_{i+t}) = x^u_g(v_{i+t}, j^p_{i+t}) \quad \forall i \in H_2, t = \{1,2\}, \forall p \in P, \forall g, f \in G \quad (7.12) \]

\[ x^r_{v,g}(i^p_r, j^p_{i+t}) = x^r_{v,g}(i^p_r, j^p_{i+t}) \quad \forall v \in V, t = \{1,2\}, \forall g, f \in G \quad (7.13) \]
where \( t \pm \tau, t \pm t' \) and \( t \pm l \) must belong to \( T \). All decision variables take only non-negative integer values. The objective function (7.1) minimizes the cost of loading, unloading, repositioning and storing empty containers over a maritime network. Using network notation, constraint set (7.2) represents flow conservation of empty containers of every type \( p \in P \) in each node \( i \in H_1 \) at every time \( t \in T \) over each scenario \( g \in G \). Constraint set (7.3) requires to assign to vessels empty containers available in each node \( i \in H_2 \) at every time \( t \in T \) over each scenario \( g \in G \). Constraint set (7.4) imposes to satisfy the demand of empty containers associated with each node \( i' \in H_2 \) at every time \( t \in T \) over each scenario \( g \in G \). Constraint set (7.5) represents flow conservation of \( p \)-type containers for the for each vessel \( k \in V \), berthing at time \( t \in T \) in port \( i \in H_1 \cup H_2 \). Constraint sets (7.6) and (7.7) ensure that inventory level of empty containers stored does not exceed a value expressed in number of containers of a given type \( \overline{p} \in P \).

Constraint set (7.8) guarantees that containers repositioned between ports does not exceed the space available for empties on vessels. Constraint sets from (7.9) to (7.15) represent the non-anticipativity conditions.

Although we take into account several container types, each scenario does not represent a pure multicommodity problem, because constraints (7.6), (7.7) and (7.8) have coefficients different from 1. Anyway a multicommodity problem, for which specific algorithms are currently available, can be easily proposed by making such modifications in the previous model:

\[
\begin{align*}
 a_{g} \cdot x_{g}^b (i_{t}^{p}, i_{t+1}^{p}) &= y_{g}^b (i_{t}^{p}, i_{t+1}^{p}) \\
 a_{g} \cdot x_{g}^b (i_{t}^{p}, j_{t+2}^{q}) &= y_{g}^b (i_{t}^{p}, j_{t+2}^{q}) \\
 a_{g} \cdot x_{k,g}^f (i_{t}^{p}, j_{t+2}^{q}) &= y_{k,g}^f (i_{t}^{p}, j_{t+2}^{q})
\end{align*}
\]
7.5 Computational results

Table 7-1 shows some computational tests performed the mathematical model described before. To solve the instances, the well-known solver Cplex-mipopt 7.5 (CPLEX optimization, 1995) is used running on a 256 MB 1.7 GHz Pentium IV computer.

<table>
<thead>
<tr>
<th>Instance</th>
<th>Number of container types</th>
<th>Number of scenarios</th>
<th>Number of Variables</th>
<th>Time for Cplex (s)</th>
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<tr>
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<td>0.53</td>
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<td>60</td>
<td>30600</td>
<td>1.21</td>
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<td>1.93</td>
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<td>459000</td>
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</tr>
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<td>0.16</td>
</tr>
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<td>10</td>
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<td>0.39</td>
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<td>0.59</td>
</tr>
<tr>
<td>P28</td>
<td>15</td>
<td>30</td>
<td>45900</td>
<td>1.26</td>
</tr>
<tr>
<td>P29</td>
<td>15</td>
<td>60</td>
<td>91800</td>
<td>2.63</td>
</tr>
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<td>15</td>
<td>90</td>
<td>137700</td>
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<td>150</td>
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<td>7.71</td>
</tr>
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<td>P32</td>
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<td>300</td>
<td>459000</td>
<td>15.35</td>
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<td>600</td>
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<td>P34</td>
<td>15</td>
<td>900</td>
<td>1377000</td>
<td>108.24</td>
</tr>
</tbody>
</table>

Table 7-1. List of instances solve by Cplex.
7.6 A decomposition algorithm for the deterministic equivalent formulation

In previous model parallel scenarios are considered together to produce a global set of decision variables. In this paragraph we consider the so-called deterministic equivalent formulation that is developed by aggregating parallel scenarios in a scenario-tree according to “progressive hedging principle” (Rockafellar and Wets, 1991). The aggregation of decisions associated with indistinguishable parts of different scenarios ensures that such decisions cannot anticipate which scenario will occur. According to Rockafellar and Wets, this decision policy is “robust”, because it provides solutions independent on information not yet available. Moreover it minimizes the risk of wrong decisions when a large set of scenarios may come true.

To perform scenario-aggregation, the definition of branching-times and stages must be presented. A branching-time represents the time-period in which some scenarios, identical up to that time, start differing. A stage indicates the sequence of time periods between two branching times. In Figure 7-3 we show a set of 8 scenarios before aggregation. Each scenario, that corresponds to a deterministic dynamic problem, is represented by a horizontal sequence of dots. The same figure illustrates the scenario-tree derived from a first branching made between periods 3 and 4 and a second one between periods 6 and 7. Stage 1 represents the characterization of the common information up to the first branch-time.

Figure 7-3. Set of 8 scenarios aggregated in a scenario-tree.
The scenario tree formulation can be described in a compact form as a collection of one deterministic model for each scenario \( g \in G \) and a set of congruity constraints for inter-stages variables. Such constraints require that decision variables must be equal among themselves up to branching-time \( \tau \) for those scenarios having a common history up to \( \tau \).

Denoting by letter \( c \) the cost vector, by \( b \) the RHS vector and by \( A \) the coefficient matrix of constraints, the resulting mathematical model can be formulated as follows:

\[
\min \sum_{g \in G} w_g c_g x_g \\
A_g x_g = b_g \quad \forall g \in G \\
x^* \in S \quad \forall g \in G
\]

where \( x^* \) represents the vector of inter-stages variables submitted to congruity constraints and \( S \) is the set of non-antipativity constraints. Figure 7-4 shows the scenario tree produced by the aggregation of parallel scenarios represented in Figure 7-2. Due to the set of information available at the decisional time, in this application the branching time is the 2\(^{nd} \) time-period of the planning horizon.

It is worth noting that the scenario tree formulation, even if huge in both variables and constraints, presents some nested algebraic structures that can be exploited to develop specialized resolution techniques. Moreover a number of exploitable structures can be produced by handling properly the scenario tree. An example of this concept is offered by the proposed application. In the first stage we add an artificial node, that is the head of all inter-stages arcs. In the second stage we introduce another artificial node that is the tail of all inter-stages arcs. Then we impose the equality of flows on arcs that originally were a single arc in the scenario tree.

This study presents a lagrangian relaxation of equal flow constraints among inter-stage arcs. This method generates several independent multicommodity problems, as shown in the Figure 7-5. Then a bundle algorithm is adopted as non differentiable optimization technique to solve the lagrangian dual (Frangioni, 1997; De Leone and Romagnoli, 1999). Such a method provides good lower bounds and may represent an alternative approach compared to standard exact algorithms. A closer-to-optimum solution can be produced by a successive heuristic phase. For example, the combined use of decomposition methods and tabu search techniques has been adopted in the context of capacititated network design problems (Crainic et al., 2000; Crainic et al., 2001).
Tables 7-2 and 7-3 provide more details about our computational experience. Often the bundle algorithm (B) converges to the optimal solution, that is verified through the solver Cplex (C), so the duality gap is 0%. Other times, the bundle algorithms returns a tight lower bound (the gap is 0.01% on average). Although Cplex outperforms the bundle algorithm in terms of resolution times, the asymptotic trend of the lagrangian dual (tables 7-4 and 7-5) suggests the opportunity of interrupting its computation after half of the current computation. Then a suitable metaheuristic can be adopted to produce an improved feasible solution for the original problem.
Figure 7-5. The scenario tree with the artificial nodes.

Table 7-2. Instances solved by the bundle algorithm with a 0% gap.
<table>
<thead>
<tr>
<th>Instance</th>
<th>Number of container types</th>
<th>Number of scenarios</th>
<th>Number of Iterations</th>
<th>Time for Bundle(s)</th>
<th>Gap Cplex(C)-Bundle(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P13</td>
<td>10</td>
<td>5</td>
<td>164</td>
<td>24.25</td>
<td>C=61200, B=61193.9</td>
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<tr>
<td>P14</td>
<td>10</td>
<td>10</td>
<td>225</td>
<td>68.84</td>
<td>C=122100, B=122091</td>
</tr>
<tr>
<td>P15</td>
<td>10</td>
<td>15</td>
<td>272</td>
<td>135.76</td>
<td>C=183300, B=183284</td>
</tr>
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<td>P25</td>
<td>15</td>
<td>5</td>
<td>230</td>
<td>52.18</td>
<td>C=90700, B=90695.9</td>
</tr>
<tr>
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<td>10</td>
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<td>15</td>
<td>338</td>
<td>267.43</td>
<td>C=271800, B=271782</td>
</tr>
</tbody>
</table>

Table 7-3. Instances solved by the bundle algorithm with a nonzero duality gap.

Table 7-4. Instance P13.

Table 7-5. Instance P25.
Chapter 8

Conclusion

Although the attention of shipping companies is mainly devoted to profitable loaded flows, empty container reposition represents a necessary phase for the continuity of their activity. Due to strong directional imbalances in trade economies, shipping companies tend to accumulate too many empty containers in import-dominant regions, involving avoidable storage costs. On the other hand, they face shortages in export-dominant areas resulting in unfulfilled transportation opportunities. A major reason for such drawbacks depends on the modest diffusion of advanced decision support systems. Therefore, the contribution of this thesis has been to develop new optimization models on this issue. Moreover, we have verified the operational implementation of modellistic solutions through several computational tests to solve real size problems in reasonable time for the shipping industry.

In Chapter 3 we have described the first dynamic optimization model to reposition empty containers between “macro-nodes” and ports. We have adopted the hour as time-period of a real size network, because it allows a more detailed representation of transportation systems than the standard daily step. Although this time-period generates large size instances, two solvers for mixed integer programming have solved several instances in short time. The use of dummy arcs has provided a friendly tool to support shipping company operations, even if they were initially introduced as a method to ensure the feasibility of the linear programming problem. When the demand of a particular container type is greater than the supply, dummy arcs have showed which transportation opportunities should be served by rented containers. On the other hand, when the supply is greater than the demand over the planning horizon, decision-makers can easily observe which containers should be given away, thus reducing shortcomings deriving from a over-large fleet.

Current logistic practices adopted to manage their empty containers have been extensively described in Chapter 4. Since the Head Office and its local agencies make decisions on empties, we have concluded that it is not possible to develop a single mathematical model which is able to capture the different aspects of real-world
applications. Therefore, in this thesis specific optimization models have been proposed to address the inland allocation (Chapter 5 and Chapter 6) and the maritime reposition (Chapter 7) of empty containers.

In Chapter 5 the street-turn strategy has been presented. It allows shipping companies to allocate containers from importers to exporters, while maximizing their profits. In order to call for the routes of trucks, we have proposed a new optimization model that adopts empty flows as unique decision variables, instead of standard routing formulations. A new formulation of costs has been proposed and tested to obtain high-quality solutions. The model has been solved exactly by Cplex in short time. Early results confirm that the proposed formulation represents a useful support for end-users in dealing with street-turns.

In Chapter 6 we have proposed two dynamic optimization models to address the local management of empty containers, when the street-turn option is not practicable. Compared to previous papers in the field, both models have provided decisions to be implemented in inland depots, while minimizing allocation costs. What is more, we have determined the number of empty containers shipped to/from ports to serve customers. Such values represent supply/demand parameters for ports in the maritime reposition issue. The first model has indicated that substitution options can significantly improve empty container allocation. However, sometimes we need to set a tolerance factor to reach optimal or near optimal solutions, depending on the adopted solver. The second model has also addressed decisions on rented containers as well. Several instances have been successfully solved by Cplex.

In Chapter 7 the maritime reposition issue has been investigated. We have proposed a dynamic optimization model, taking into account uncertain parameters through a set of representative scenarios. A weight can be assigned to each scenario to characterize its relative importance, when reliable probabilistic rules cannot be adopted. Several instances have been efficiently solved by Cplex in a time suitable for the operating needs of the shipping industry. Then the associated deterministic equivalent formulation has been taken into account, a Lagrangian relaxation of inter-stages constraints has been proposed and a freeware bundle algorithm had been adopted to solve the Lagrangian dual. Early findings of this research have shown an asymptotic convergence of the
bundle algorithm to good lower bounds. Research is still in progress to exploit better algebraic structures.

To conclude, a part of the methods described in this thesis has been transferred to the industry. The continuing relationship with some shipping companies is still retaining our attention to develop more adherent methodologies to support their complex activity. At the moment some industrial projects have been undertaken.
References.


http://www.informare.it

http://www.mol.co.jp

http://www.pmanet.org

http://www.tbridge.it


